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A GENERAL PERTURBATIONS DIFFERENTIAL CORRECTION PROGRAM

J. L. Arsenault
J. R. Kuhlman
L. W. Stumpf

Aeronutronic
A Division of Ford Motor Company
Newport Beach, California

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Technical Documentary Report No. ESD-TDR-63-432
1 August 1963
Contract AF 19(628)-562

Prepared for:
496L Systems Project Office
Electronic Systems Division
Air Force Systems Command
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Laurence G. Hanscom Field
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ABSTRACT

An experimental computer program is described, which calculates Earth Satellite ephemerides, corrects orbit elements and evaluates the effects of various terms of the bulge perturbation theory. Perturbations by solar radiation pressure and atmospheric drag are also represented. The differential correction employs a weighted least-squares reduction. Formulation, flow charts, input formats and sample cases are given.

FOREWORD

The authors wish to acknowledge the assistance of Mr. Kenneth Stewart who reprogrammed parts of the operational differential correction program (SGPDC)* for use in this program.

* Aeronutronic publication U-1691, revised 1 October 1962, pp. 3-61 to 3-95.

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SECTION 1

INTRODUCTION

The Experimental General Perturbations Differential Correction Program calculates the ephemeris of an Earth satellite by a General Perturbations technique and improves the orbital elements by a differential correction process using satellite observations. The ephemeris calculation includes the analytical expressions of the perturbations caused by the asphericity of the Earth and of the effects due to direct solar radiation pressure on a close-Earth satellite. Frictional effects due to the Earth's atmosphere are determined empirically from observational data.

The program is equipped to compare the complete first-order asphericity theory with simplified theories, in which selected terms are omitted. The comparison is made in the magnitude and in the radial, transverse and orthogonal components of the displacement from the position obtained with the complete theory.

The effects of direct solar radiation on the orbit of an Earth satellite are introduced through the perturbations in the orbital parameters:

$e \cos \pi$

$e \sin \pi$

L, mean longitude

$\sin i \sin \Omega$

and $\sin i \cos \Omega$

where $\pi = \Omega + \omega$ (see also Figure 3)

Ω is the longitude of the ascending node

ω is the argument of perigee

e is the eccentricity of orbit

and i is the inclination of the orbit-plane to the equator-plane.

This set of elements defines nearly-circular orbits without the difficulties caused by the singularities of the classical elements at zero eccentricity. Only the long-period changes of the parameters are retained from the development.* The effects on these long-period terms of the satellite being eclipsed by the Earth in part of its orbit are also included.

The perturbation theory for the asphericity of the Earth contains first-order short-period terms and second-order secular and long-period terms. The second zonal harmonic coefficient, J_2, of the geopotential function is considered to be of the first order. The third and fourth zonal harmonic coefficients, J_3 and J_4, respectively, as well as J^2 are of the second order. For a more detailed analysis, reference is made to Aeronutronic Report S-981. In addition, the works of Brouwer** and Kozai*** are useful for comparison of the results. The second-order secular and long-period terms in the mean anomaly, M, were adopted from Brouwer's analysis.

The expressions presented in the formulation of these bulge perturbations are free of low-eccentricity singularities due to the choice of the parameters $e \sin \omega$, $e \cos \omega$ and the argument of latitude, u, instead of the corresponding classical elements. The short-period terms in the radial distance, r, and the argument of latitude, u, are used rather than the related expressions for the semi-axis major, a, the eccentricity, e, the argument of perigee, ω, and the mean anomaly, M, which contain $\frac{1}{e}$. Long-period terms are calculated for the elements $e \cos \omega$, $e \sin \omega$, and L since the classical parameters also have terms containing J_3/e as a coefficient.

---

* Koskela, Paul, 1961, "Orbital Effects of Solar Radiation Pressure on an Earth Satellite," Aeronutronic Publication No. U-1357.

** Brouwer, Dirk, 1959, Astronomical Journal, 64, 378.

*** Kozai, Y, 1959, loc. cit, 367.

Several of the bulge perturbation terms contain the factor

$$e^{-2}\,(1 - \sqrt{1 - e^2}),$$

which approaches ½ as a limit at zero eccentricity. It is approximated by a rapidly converging series expansion when e is small, as are two similar coefficients.

The expression $(4\text{-}5 \sin^2 i)$ appears in the denominator of many of the long-period terms shown in the formulation. This quantity produces a singularity at the critical inclination angle of 63.4349...degrees. In this program the terms containing $(4\text{-}5 \sin^2 i)$ are automatically set equal to zero when the inclination approaches the critical value.*

The perturbation theory is based on the Gaussian form of the expressions for the variations of the parameters; that is, three mutually perpendicular components of the perturbing acceleration are used. The expressions are developed in terms of the true anomaly and thus have no remainder terms in powers of the eccentricity.

Section 2 of this report lists the equations necessary for ephemeris calculation. The equations are presented in a form which allows clear identification of each bulge term for the experimentation. The use of low-eccentricity orbital parameters, the inclusion of the effects of the Earth's bulge on the satellite's velocity, and the separation of the terms by size with regard to the order of the harmonic coefficients and the power of the eccentricity account for the large number of terms.

Section 3 describes the computer program which has resulted from this project. The ephemeris calculation consists of determining the perturbative changes in the selected orbital parameters or coordinates and calculating the geocentric position and velocity vectors at desired times. The differential correction process obtains the correction to any or all of the six orbital elements: U_o, the mean argument of latitude, $a_{xN} = e \cos \omega$, $a_{yN} = e \sin \omega$, n , the orbital mean motion, i and Ω ; and the correction to the drag parameters c" and d, if desired. The program has the capacity to weight the observations according to the given value of the standard deviation of each observation, be it range, azimuth, elevation angle, right ascension, declination or range-rate. It may also consider the weights assigned to an observing station for the quality of its observational data.

---

* Aeronutronic Report U-1672, "Numerical Values of General Perturbations Terms in Orbital Parameters," by J. L. Arsenault and its appendix explain the justification for this. Basically, as the rate of change in the argument of perigee approaches zero, these particular long-period terms become of very long period and act like secular terms. The mean value of the terms can be successfully combined with the mean value of the relevant parameter.

SECTION 2

FORMULATION

Because the treatment of perturbations due to direct solar radiation in this program is somewhat novel, the formulation is detailed here. The formulation of all the ephemeris calculation, including the other perturbations and of the differential correction are then given.

2.1 DIRECT SOLAR RADIATION PERTURBATIONS

The magnitude of the force acting on a satellite due to direct solar radiation is

$$F_{\odot} = |\underline{F}_{\odot}| = \gamma \nu P_{\odot} A$$

where

A is the effective cross-sectional area of the satellite to radiation pressure

$P_{\odot}$ is the solar radiation pressure in the vicinity of the Earth, assumed constant

γ is a factor depending on the reflecting characteristics of the satellite's surface

ν is an eclipse factor to be considered in detail later (Section 2.1.3)

It is customary to assume that the direction and magnitude of this force is constant along the entire orbit, except in the Earth's shadow, where it does not exist.

2.1.1 Sun's True Longitude

The Sun's true longitude, $\ell_\odot$, at time t is computed from

$$\ell_\odot = (L_\odot)_o + n_\odot(t-t_o) + 2\, e_\odot \sin M_\odot + \frac{5}{4} e_\odot^2 \sin 2\, M_\odot + \ldots \text{(in radians)}$$

where

$(L_\odot)_o$ is the Sun's mean longitude at some epoch, t_o

$n_\odot$ = $0^\circ.9856$/day, the Sun's mean daily motion

$e_\odot$ = 0.016726, the eccentricity of the Earth's orbit

$M_\odot$ = $(L_\odot)_o + n_\odot\,(t-t_o) - \Pi_\odot$

$\Pi_\odot$ = $282^\circ.253$

2.1.2 Perturbations in the Elements Due to Direct Solar Radiation

Compute the perturbations in the elements a, $(e \cos\pi)$, $(e \sin\pi)$, W_x, W_y, and L due to direct solar radiation. The required expressions are derived in Aeronutronic Report U-1357. However, in the results which follow, $\frac{d\Omega}{dE}$ and $\frac{d\omega}{dE}$ have been replaced by their variations with respect to the modified time variable, τ, by means of

$$\frac{d\Omega}{dE} = \frac{k_e}{n}\, \Omega$$

and

$$\frac{d\omega}{dE} = \frac{k_e}{n}\, \omega$$

The mean motion, n, has been replaced by $n = k_e \sqrt{\mu}\ /a^{3/2}$ radians/min.

The a_i coefficients are listed in Table I. The semi-major axis a, is in units of Earth radii; $\mu = 1\ \frac{(\text{Earth radius})^3}{(k_e^{-1}\ \text{min})^2}$; the satellite's

mass, m, is in grams; the magnitude of the force due to direct solar radiation is $F_{\odot} = \nu \gamma P_{\odot} A$; the eclipse factor υ and the reflectivity γ are dimensionless; the satellite's effective cross-sectional area, A, is in cm^2; and the solar radiation pressure is $P_{\odot} = 4.592 \times 10^{-8} \frac{gm}{cm^2} \frac{radii}{(k_e^{-1} min)^2}$

(a) In the semi-major axis there are only short-period perturbations due to direct solar radiation. Therefore, let

$$\Delta a = 0$$

Define an unperturbed ~~semi-axis~~ major:

$$a_{up} = a_o \left[1 + \frac{3}{2} \frac{J_2 a_e^2}{p_o^2} \sqrt{1-e_o^2} \left(1 - 3/2 \sin^2 i_o\right) \right]$$

(b) Calculate $\Delta (e \cos \pi)$:

$$\Delta (e \cos \pi) = - \frac{3}{16} \sqrt{a_{up} (1-e_o^2)} \frac{F_{\odot}}{\sqrt{\mu} m} \Big\{ 4 \sin i_o \sin \epsilon \left[- a_{11} \cos (\Omega + \ell_{\odot}) + a_{12} \cos (\Omega - \ell_{\odot}) \right]$$
$$- 2 (1 - \cos i_o) \left[a_{13} (1 - \cos \epsilon) \cos (2\Omega + \ell_{\odot}) + a_{14} (1 + \cos \epsilon) \cos (2\Omega - \ell_{\odot}) \right]$$
$$-4 a_{15} (1 + \cos i_o) (\cos \epsilon) \cos \ell_{\odot} \Big\}$$
$$- \frac{3}{16} \frac{\sqrt{a_{up} e_o^2}}{\sqrt{1-e_o^2}} \frac{\sin i_o}{(1 + \cos i_o)} \frac{F_{\odot}}{\sqrt{\mu} m} \Big\{ 2 \cos i_o \sin \epsilon \left[- a_{11} \cos (\Omega + \ell_{\odot}) \right.$$
$$+ a_{12} \cos (\Omega - \ell_{\odot}) + a_{16} \cos (\Omega + 2\omega + \ell_{\odot})$$
$$\left. - a_{17} \cos (\Omega + 2\omega - \ell_{\odot}) \right]$$

$$+ \sin i_o (1 + \cos \epsilon) [- a_{14} \cos (2\Omega - \ell_\bullet)$$

$$- a_{18} \cos (2\omega + \ell_\bullet) + a_{20} \cos (2\Omega + 2\omega - \ell_\bullet)]$$

$$+ \sin i_o (1 - \cos \epsilon) [- a_{13} \cos (2\Omega + \ell_\bullet)$$

$$- a_{19} \cos (2\omega - \ell_\bullet) + a_{21} \cos (2\Omega + 2\omega + \ell_\bullet)]$$

$$+ 2 a_{15} \sin i_o \cos \epsilon \cos \ell_\bullet \Big\}$$

(c) Calculate $\Delta (e \sin \pi)$:

$$\Delta (e \sin \pi) = - \frac{3}{16} \sqrt{a_{u_p}(1-e_o^2)} \frac{F_\bullet}{\sqrt{\mu m}} \Big\{ 4 \sin i_o \sin \epsilon$$

$$[-a_{11} \sin (\Omega + \ell_\bullet) + a_{12} \sin (\Omega - \ell_\bullet)]$$

$$- 2 (1 - \cos i_o) [a_{13} (1 - \cos \epsilon) \sin (2\Omega + \ell_\bullet)$$

$$+ a_{14} (1 + \cos \epsilon) \sin (2\Omega - \ell_\bullet)]$$

$$- 4 a_{15} (1 + \cos i_o) \sin \ell_\bullet \Big\}$$

$$- \frac{3}{16} \frac{\sqrt{a_{u_p}}\, e_o^2}{\sqrt{1 - e_o^2}} \frac{\sin i_o}{(1 + \cos i_o)} \frac{F_\bullet}{\sqrt{\mu m}} \Big\{ 2 \cos i_o \sin \epsilon$$

$$[- a_{11} \sin (\Omega + \ell_\bullet)$$

$$+ a_{12} \sin (\Omega - \ell_\bullet) + a_{16} \sin (\Omega + 2\omega + \ell_\bullet)$$

$$- a_{17} \sin (\Omega + 2\omega - \ell_\bullet)]$$

$$+ \sin i_o (1 + \cos \epsilon) [- a_{14} \sin (2\Omega - \ell_\bullet)$$

$$- a_{18} \sin (2\omega + \ell_\bullet) + a_{20} \sin (2\Omega + 2\omega - \ell_\bullet)]$$

$$+ \sin i_o (1 - \cos \epsilon) [- a_{13} \sin (2\Omega + \ell_\odot) - a_{19} \sin (2\omega - \ell_\odot) + a_{21} \sin (2\Omega + 2\omega + \ell_\odot)] + 2 a_{15} \sin i_o \sin \ell_\odot \Big\}$$

(d) Calculate ΔW_x:

$$\Delta W_x = \frac{3}{16} \frac{\sqrt{a_{u_p}}\, e_o}{\sqrt{1 - e_o^2}} \frac{F_\odot}{\sqrt{\mu m}} \Big\{ 2 \cos i_o \sin \epsilon (1 + \cos i_o) [- a_1 \sin (\Omega + \omega + \ell_\odot) + a_2 \sin (\Omega + \omega - \ell_\odot)]$$

$$+ 2 \cos i_o \sin \epsilon (1 - \cos i_o) [a_3 \sin (\Omega - \omega + \ell_\odot) - a_4 \sin (\Omega - \omega - \ell_\odot)]$$

$$+ 2 a_5 \sin i_o (\cos i_o + \cos \epsilon) \sin (\omega + \ell_\odot)$$

$$+ 2 a_6 \sin i_o (\cos i_o - \cos \epsilon) \sin (\omega - \ell_\odot)$$

$$+ a_7 \sin i_o (1 - \cos i_o) (1 + \cos \epsilon) \sin (2 \Omega - \omega - \ell_\odot)$$

$$+ a_8 \sin i_o (1 - \cos i_o) (1 - \cos \epsilon) \sin (2 \Omega - \omega + \ell_\odot)$$

$$- a_9 \sin i_o (1 + \cos i_o) (1 + \cos \epsilon) \sin (2\Omega + \omega - \ell_\odot)$$

$$- a_{10} \sin i_o (1 + \cos i_o) (1 - \cos \epsilon) \sin (2\Omega + \omega + \ell_\odot) \Big\}$$

(e) Calculate ΔW_y :

$$\Delta W_y = \frac{3}{16} \frac{\sqrt{a_{u_p}}\, e_o}{\sqrt{1 - e_o^2}} \frac{F_\odot}{\sqrt{\mu m}} \Big\{ 2 \cos i_o \sin \epsilon (1 + \cos i_o) [a_1 \cos (\Omega + \omega + \ell_\odot) - a_2 \cos (\Omega + \omega - \ell_\odot)]$$

$$+ 2 \cos i_o \sin \epsilon (1 - \cos i_o) [- a_3 \cos (\Omega - \omega + \ell_\bullet) + a_4 \cos (\Omega - \omega - \ell_\bullet)]$$

$$- 2 a_5 \sin i_o (1 + \cos i_o \cos \epsilon) \cos (\omega + \ell_\bullet)$$

$$- 2 a_6 \sin i_o (1 - \cos i_o \cos \epsilon) \cos (\omega - \ell_\bullet)$$

$$- a_7 \sin i_o (1 - \cos i_o) (1 + \cos \epsilon) \cos (2\Omega - \omega - \ell_\bullet)$$

$$- a_8 \sin i_o (1 - \cos i_o) (1 - \cos \epsilon) \cos (2\Omega - \omega + \ell_\bullet)$$

$$+ a_9 \sin i_o (1 + \cos i_o) (1 + \cos \epsilon) \cos (2\Omega + \omega - \ell_\bullet)$$

$$+ a_{10} \sin i_o (1 + \cos i_o) (1 - \cos \epsilon) \cos (2\Omega + \omega + \ell_\bullet) \Big\}$$

(f) Calculate ΔL:

$$\Delta L = - \frac{3}{4}\sqrt{a_{u_p}} e_o \left[1 - \frac{\sqrt{1-e_o^2}}{2(1+\sqrt{1-e_o^2})} \right] \frac{F_\bullet}{\sqrt{\mu m}} \Big\{ a_1 (1+\cos i_o)(1-\cos \epsilon) \sin (\Omega + \omega + \ell_\bullet)$$

$$+ a_2 (1 + \cos i_o) (1 + \cos \epsilon) \sin (\Omega + \omega - \ell_\bullet)$$

$$+ a_3 (1 - \cos i_o) (1 - \cos \epsilon) \sin (\Omega - \omega + \ell_\bullet)$$

$$+ a_4 (1 - \cos i_o) (1 + \cos \epsilon) \sin (\Omega - \omega - \ell_\bullet)$$

$$+ 2 \sin i_o \sin \epsilon [- a_5 \sin (\omega + \ell_\bullet) + a_6 \sin (\omega - \ell_\bullet)] \Big\}$$

$$+ \frac{3}{8} \frac{\sqrt{a_{u_p}} e_o}{\sqrt{1-e_o^2}} \frac{\sin i_o}{(1 + \cos i_o)} \frac{F_\bullet}{\sqrt{\mu m}} \Big\{ \sin i_o (1 - \cos \epsilon) [-a_1 \sin (\Omega + \omega + \ell_\bullet) + a_3 \sin (\Omega - \omega + \ell_\bullet)]$$

$$+ \sin i_o (1 + \cos \epsilon) [a_4 \sin (\Omega - \omega - \ell_\bullet) - a_2 \sin (\Omega + \omega - \ell_\bullet)]$$

$$+ 2 \cos i_o \sin \epsilon [- a_5 \sin (\omega + \ell_\bullet) + a_6 \sin (\omega - \ell_\bullet)] \Big\}$$

TABLE I

The Coefficients $a_i \left[\text{radians}/k_e^{-1}\ \text{min}\right]^{-1}$

$$a_1 = (\dot{\Omega} + \dot{\omega} + n_\odot)^{-1}$$
$$a_2 = (\dot{\Omega} + \dot{\omega} - n_\odot)^{-1}$$
$$a_3 = (\dot{\Omega} - \dot{\omega} + n_\odot)^{-1}$$
$$a_4 = (\dot{\Omega} - \dot{\omega} - n_\odot)^{-1}$$
$$a_5 = (\dot{\omega} + n_\odot)^{-1}$$
$$a_6 = (\dot{\omega} - n_\odot)^{-1}$$
$$a_7 = (2\dot{\Omega} - \dot{\omega} - n_\odot)^{-1}$$
$$a_8 = (2\dot{\Omega} - \dot{\omega} + n_\odot)^{-1}$$
$$a_9 = (2\dot{\Omega} + \dot{\omega} - n_\odot)^{-1}$$
$$a_{10} = (2\dot{\Omega} + \dot{\omega} + n_\odot)^{-1}$$
$$a_{11} = (\dot{\Omega} + n_\odot)^{-1}$$
$$a_{12} = (\dot{\Omega} - n_\odot)^{-1}$$
$$a_{13} = (2\dot{\Omega} + n_\odot)^{-1}$$
$$a_{14} = (2\dot{\Omega} - n_\odot)^{-1}$$
$$a_{15} = (n_\odot)^{-1}$$
$$a_{16} = (\dot{\Omega} + 2\dot{\omega} + n_\odot)^{-1}$$
$$a_{17} = (\dot{\Omega} + 2\dot{\omega} - n_\odot)^{-1}$$
$$a_{18} = (2\dot{\omega} + n_\odot)^{-1}$$
$$a_{19} = (2\dot{\omega} - n_\odot)^{-1}$$
$$a_{20} = (2\dot{\Omega} + 2\dot{\omega} - n_\odot)^{-1}$$
$$a_{21} = (2\dot{\Omega} + 2\dot{\omega} + n_\odot)^{-1}$$

Note: $n_\odot = 1.6064 \times 10^{-4} \dfrac{\text{radians}}{k_e^{-1}\ \text{min}}$

2.1.3 Eclipse Factor

When a satellite is continuously exposed to sunlight, as Echo was from its launch on 12 August 1960, to 24 August 1960, the perturbing force due to solar radiation is continuous and the eclipse factor is $\nu = 1$ in the expression for $F_{\odot}$. The integration of the perturbation equations presents no problem in this case. More often, however, the satellite passes through the Earth's shadow (where $\nu = 0$) on each revolution. The eclipse thereby produces discontinuities in the perturbing force. The customary way of handling this discontinuity is to evaluate the perturbative variations in the elements after each revolution. The values of the satellite's anomaly at the points where the satellite leaves and enters the shadow on each revolution are then the lower and upper limits of integration, respectively, in the perturbation equations. The quantities Ω, ω, $\ell_{\odot}$, and the eclipse points are considered constant during the one revolution interval. It is desirable to avoid this once-per-revolution integration, however, and to integrate over a much longer time interval.

The time spent in eclipse changes continually due to the apparent motion of the Sun and the perturbations on the satellite orbit. The change per revolution is most rapid when the satellite orbit plane is entering or leaving the Earth's shadow. Once in the shadow, the time spent in eclipse per revolution does not change appreciably over a number of days. This is illustrated in Figure 1, which shows the variation of time in sunlight per revolution for Echo around 24 August 1960, when it was ending a period of some 12 days of continuous exposure to sunlight. The change was 10 percent the first day, not quite 5 percent the second day, and it quickly decreased to 1 percent per day and less. This suggests the following approximation, which was used in integrating the perturbation equations over many revolutions.

The parameters were integrated over many revolutions on the basis of continuous exposure to sunlight. The eclipse factor ν appearing in the expression for $F_{\odot}$ was then given the form

$$\nu = 1 - \frac{\Delta t}{P}$$

where Δt is the time per revolution spent in the shadow where the perturbing force does not act, and P is the period. During the intervals when Δt is changing very slowly, ν need be re-evaluated only every day or so.

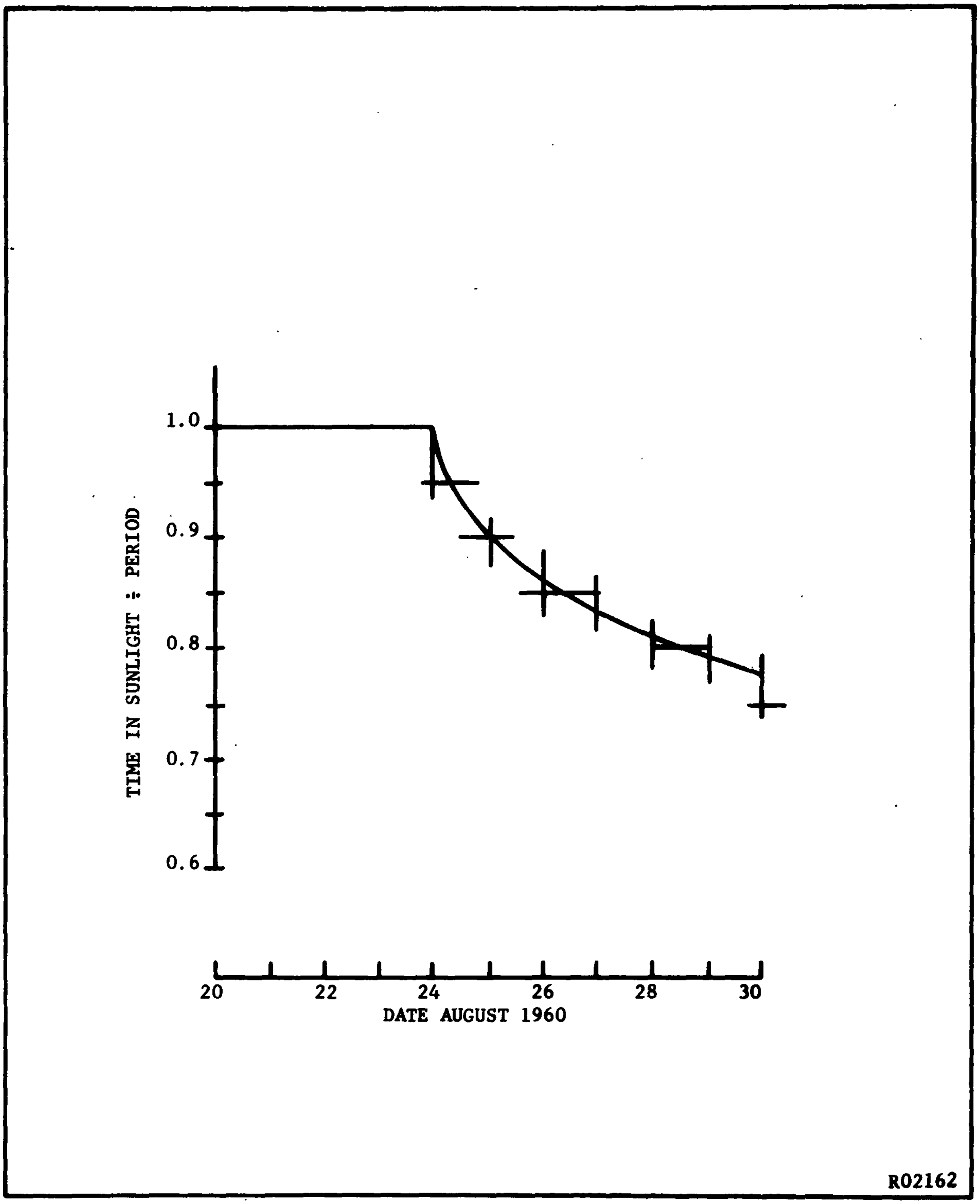

FIGURE 1. FRACTION OF TIME SPENT IN SUNLIGHT BY ECHO I

The eclipse factor ν is determined according to the following steps. If the time t is within 24 hours of the time t_i for which the last eclipse factor ν_i was computed, this section is bypassed.

(a) Calculate the geocentric angular distance d_s between the orbit plane and the shadow axis (see Figure 2):

$$d_s = \sin^{-1} (- \underline{W} \cdot \underline{L}_{\odot}) = \sin^{-1} (- W_x L_{x\odot} - W_y L_{y\odot} - W_z L_{z\odot})$$

where

$$\underline{W} \begin{cases} W_x = \sin \Omega \sin i \\ W_y = - \cos \Omega \sin i \\ W_z = \cos i \end{cases}$$

$$\underline{L}_o \begin{cases} L_{xo} = \cos \ell_{\odot} \\ L_{yo} = \cos \epsilon \sin \ell_{\odot} \\ L_{zo} = \sin \epsilon \sin \ell_{\odot} \end{cases}$$

The angle d_s is between $\pm 90^o$. It is taken as positive when the shadow axis is north of the orbit plane and negative if south of the orbit plane.

(b) Calculate the angle

$$(\pi - \psi) = \sin^{-1} \frac{a_e}{a}$$

If $| \pi - \psi | - | d_s | < 0.01$, assume $\nu = 1$, that is, the satellite will not pass through the shadow, and omit the following sequence of equations for determining ν.

If $| \pi - \psi | - | d_s | > 0.01$, the eccentric anomalies at the points where the satellite enters, E_{in}, and leaves, E_{out}, the Earth's shadow must be calculated.

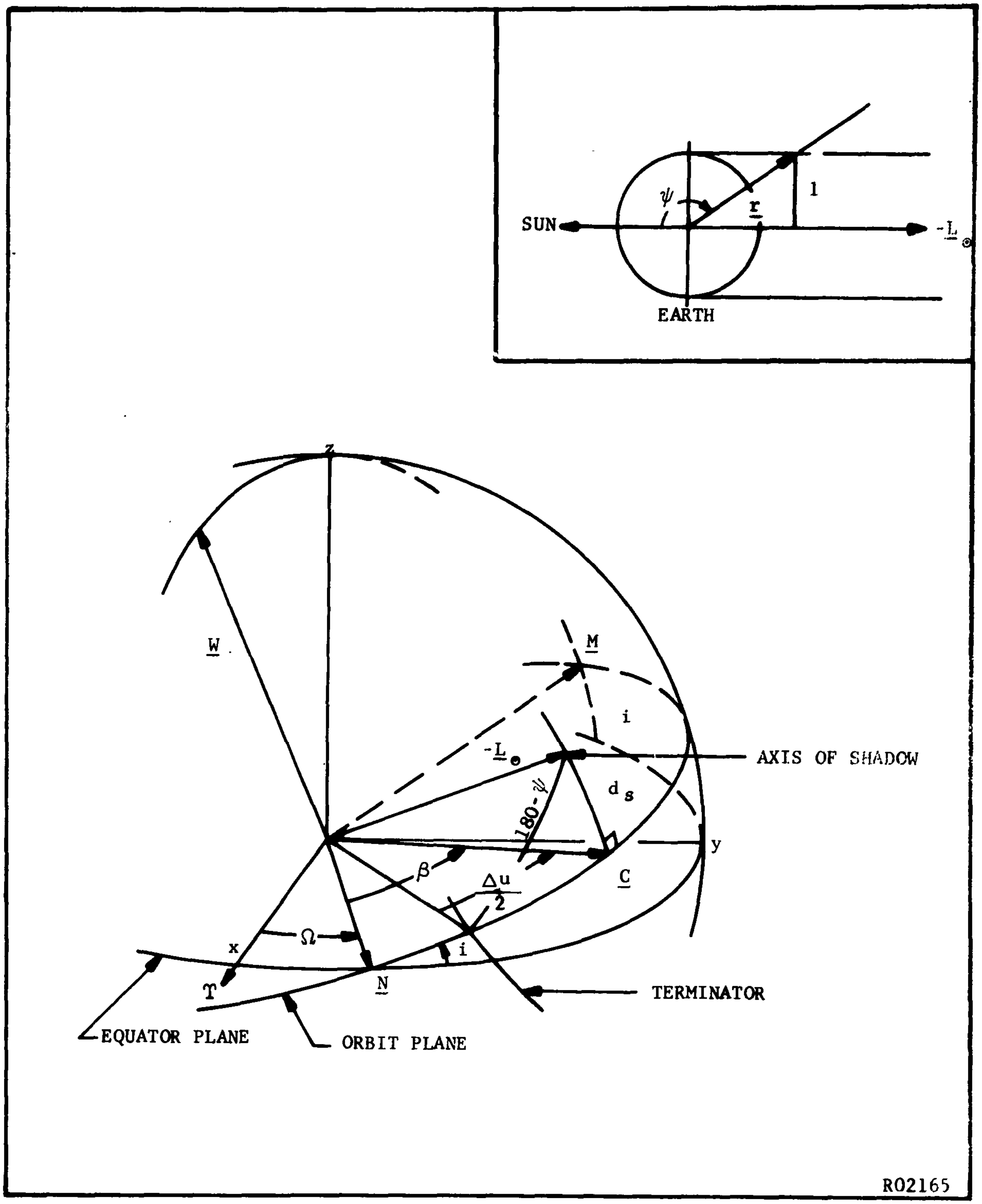

FIGURE 2. ECLIPSE GEOMETRY PROJECTED ONTO CELESTIAL SPHERE

(c) The values of the eccentric anomaly at the eclipse points are the solutions of

$$1 - a^2 (1 - e \cos E)^2 + a^2 \left[(\cos E - e)(\underline{P} \cdot \underline{L}_\odot) + \sqrt{1 - e^2} \sin E (\underline{Q} \cdot \underline{L}_\odot)\right]^2 = 0 .$$

There are only two valid solutions to this equation since the bracketed quantity must be negative in order for an eclipse to take place.

Calculate initial estimates for E at the beginning (E_{in}) and end (E_{out}) of eclipse:

$$E_{in} = \beta - \frac{\Delta u}{2} - \omega$$

and

$$E_{out} = \beta + \frac{\Delta u}{2} - \omega$$

where

$$\Delta u = 2 \cos^{-1} \left[\frac{- \cos \psi}{\cos d_s}\right] \quad ; \Delta u < 180^\circ$$

$$\beta = \tan^{-1} \frac{\underline{C} \cdot \underline{M}}{\underline{C} \cdot \underline{N}}$$; the quadrant is determined from the signs of the numerator and denominator.

The nodal unit vectors $\underline{N}$ and $\underline{M}$ are shown in Figure 3. Their components are

$$\underline{N} \begin{cases} N_x = \cos \Omega \\ N_y = \sin \Omega \\ N_z = 0 \end{cases}$$

and

$$\underline{M} \begin{cases} M_x = - \sin \Omega \cos i \\ M_y = \cos \Omega \cos i \\ M_z = \sin i \end{cases}$$

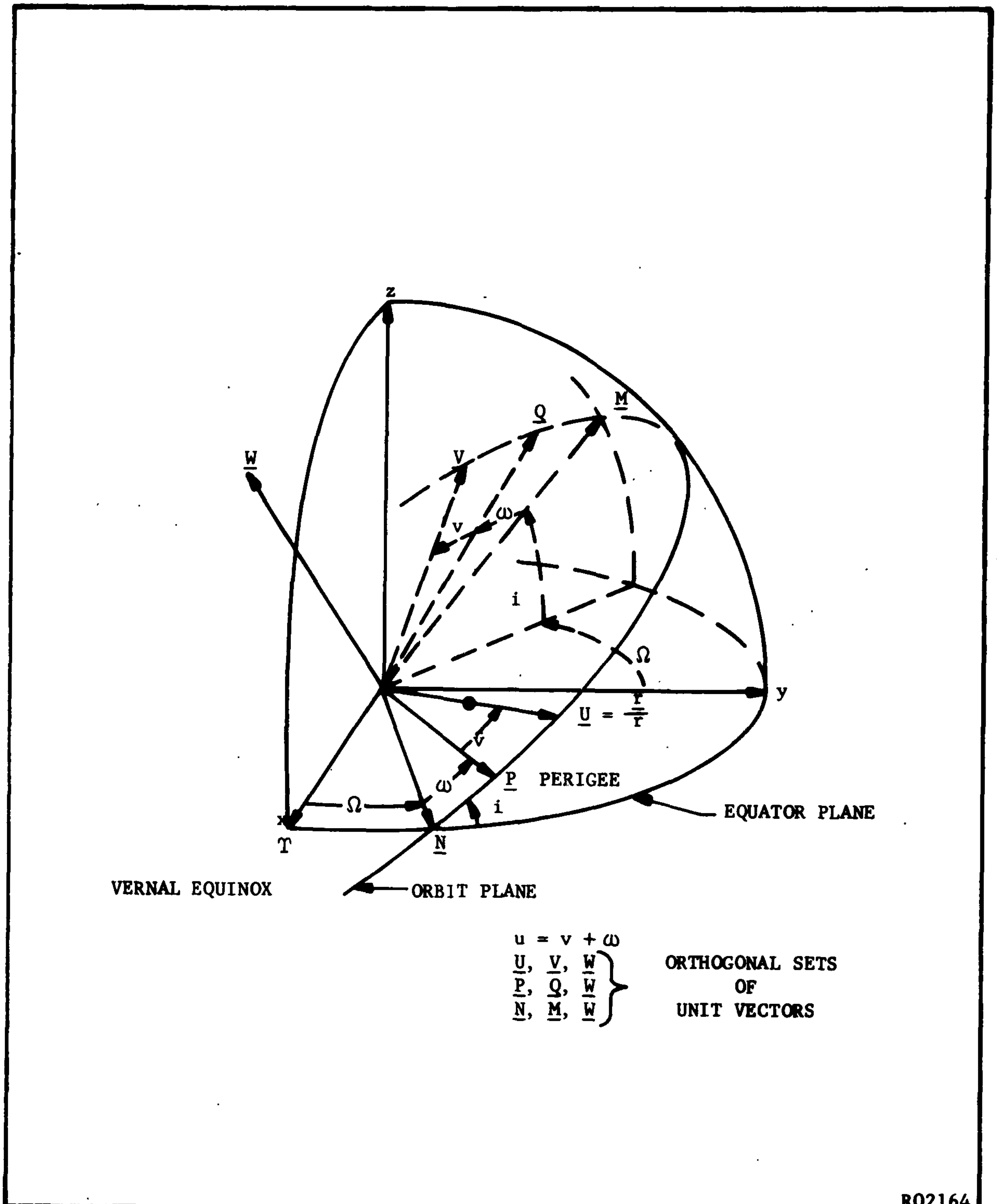

FIGURE 3. ORIENTATION UNIT VECTORS AND ANGLES

and the auxiliary unit vector $\underline{C}$ is (cf. Figure 2)

$$\underline{C} = - \sec d_s \underline{L}_{\odot} - \tan d_s \underline{W} \quad .$$

(d) Calculate the dot products $\underline{P} \cdot \underline{L}_{\odot}$ and $\underline{Q} \cdot \underline{L}_{\odot}$:

$$\begin{aligned} \underline{P} \cdot \underline{L}_{\odot} = \frac{1}{4} \Big\{ & (1 - \cos i)\,(1 + \cos \epsilon)\,\cos(\Omega - \omega - \ell_{\odot}) \\ + & (1 - \cos i)\,(1 - \cos \epsilon)\,\cos(\Omega - \omega + \ell_{\odot}) \\ + & (1 + \cos i)\,(1 + \cos \epsilon)\,\cos(\Omega + \omega - \ell_{\odot}) \\ + & (1 + \cos i)\,(1 - \cos \epsilon)\,\cos(\Omega + \omega - \ell_{\odot}) \\ + & 2 \sin i \sin\epsilon\,[\cos(\omega - \ell_{\odot}) - \cos(\omega + \ell_{\odot})] \Big\} \end{aligned}$$

$$\begin{aligned} \underline{Q} \cdot \underline{L}_{\odot} = \frac{1}{4} \Big\{ & (1 - \cos i)\,(1 + \cos \epsilon)\,\sin(\Omega - \omega - \ell_{\odot}) \\ + & (1 - \cos i)\,(1 - \cos \epsilon)\,\sin(\Omega - \omega + \ell_{\odot}) \\ - & (1 + \cos i)\,(1 + \cos \epsilon)\,\sin(\Omega + \omega - \ell_{\odot}) \\ - & (1 + \cos i)\,(1 - \cos \epsilon)\,\sin(\Omega + \omega + \ell_{\odot}) \\ + & 2 \sin i \sin\epsilon\,[-\sin(\omega - \ell_{\odot}) + \sin(\omega + \ell_{\odot})] \Big\} \end{aligned}$$

(e) With the initial estimates of E_{in} and E_{out}, f(E) and f'(E) are calculated for use with Newton's approximation method. Calculate

$$\begin{aligned} f(E) = 1 - a^2 (1 - e \cos E)^2 + a^2 \Big[& (\cos E - e)\,(\underline{P} \cdot \underline{L}_{\odot}) \\ & + \sqrt{1 - e_o^2}\, \sin E\,(\underline{Q} \cdot \underline{L}_{\odot}) \Big]^2 \end{aligned}$$

and

$$\begin{aligned} f'(E) = & - 2 a^2 e \sin E\,(1 - e \cos E) \\ & + 2 a^2 \left[(\cos E - e)\,(\underline{P} \cdot \underline{L}_{\odot}) + \sqrt{1 - e_o^2}\, \sin E\,(\underline{Q} \cdot \underline{L}_{\odot}) \right] \\ & \times \left[- \sin E\,(\underline{P} \cdot \underline{L}_{\odot}) + \sqrt{1 - e_o^2}\, \cos E\,(\underline{Q} \cdot \underline{L}_{\odot}) \right] . \end{aligned}$$

(f) Calculate the correction to the initial estimates of E_{in} and E_{out}:

$$\Delta E = \frac{-f(E)}{f'(E)}$$

This correction is applied to the preceding value, $E_{n+1} = E_n + \Delta E$. The iteration is continued until

$$\left| \frac{\Delta E}{E} \right| \leq 0.01$$

(g) The eccentric anomaly at the eclipse points are used to calculate the mean anomaly at these points from

$$M_{in} = E_{in} - e \sin E_{in}$$

and

$$M_{out} = E_{out} - e \sin E_{out}$$

These in turn are used to find the time spent in the shadow:

$$\Delta t = \frac{M_{out} - M_{in}}{n}$$

from which the eclipse factor ν is

$$\nu = 1 - \frac{\Delta t}{P_{a_o}} .$$

where

$$P_{a_o} = \frac{2\pi}{n_o} \left[1 - \frac{3}{2} J_2 \left(\frac{a_e}{p_o} \right)^2 \left(1 - \frac{3}{2} \sin^2 i_o\right) \left(1 - \frac{1}{2} e_o^2\right) \right]$$

2.2 EPHEMERIS CALCULATION

The ephemeris calculation determines a satellite's geocentric position $\underline{r}$ and velocity $\dot{\underline{r}}$ at a given time, t, from the initial orbit elements a_{xN_o}, a_{yN_o}, $\underline{h}_o$, L_o, and the drag coefficients c_o, and d_o, at an

epoch time t_o. The perturbations caused by the Earth's atmosphere, by direct solar radiation (when significant), and by the asphericity of the Earth (including terms due to the second, third, and fourth zonal harmonics) are included in this calculation.

2.2.1 <u>Preliminary Computations</u>

The following quantities, required in later calculations, are calculated from the given initial conditions (see section 3.1.1).

(a) Compute the semi-latus rectum p_o, the eccentricity, e_o, the mean semi-major axis a_o, and the distance to perigee q_o:

$$p_o = \underline{h}_o \cdot \underline{h}_o = h_{x_o}^2 + h_{y_o}^2 + h_{z_o}^2$$

$$e_o^2 = a_{xN_o}^2 + a_{yN_o}^2$$

$$a_o = p_o / (1 - e_o^2)$$

$$q_o = a_o (1 - e_o)$$

(b) Compute the three orientation angles: i_o, the inclination; Ω_o, the longitude of the ascending node; and ω_o, the argument of perigee:

$$i_o = \cos^{-1} \frac{h_{z_o}}{\sqrt{p_o}}; \quad 0 \leq i_o < \pi$$

$$\Omega_o = \tan^{-1} \frac{h_{x_o}}{-h_{y_o}}; \quad \text{the quadrant is determined from the signs of the numerator and denominator, } 0 \leq \Omega_o < 2\pi$$

$$\omega_o = \tan^{-1} \frac{a_{yN_o}}{a_{xN_o}}; \quad \text{if } e_o \geq .001 \text{ ; } 0 \leq \omega_o < 2\pi$$

$$\text{if } e_o < .001 \text{ , } \omega_o = 0$$

(c) Compute the mean argument of latitude, U_o, and the mean value of the mean motion, n_o:

If $h_{z_o} \geq 0$ (direct motion)

$$U_o = L_o - \Omega_o$$

If $h_{z_o} < 0$ (retrograde motion)

$$U_o = L_o + \Omega_o$$

$$n_o = \frac{k_e\sqrt{\mu}}{a_o^{3/2}} \left[1 - \frac{3}{4} J_2 \frac{a_e^2}{p_o^2} \sqrt{1 - e_o^2} \; (1 - \frac{3}{2} \sin^2 i_o)\right]$$

where

$$\frac{3}{2} J_2 a_e^2 = 1.623675 \times 10^{-3} \qquad \text{(earth radii)}^2$$

and

$$k_e\sqrt{\mu} = 0.074\ 365\ 74 \qquad \text{(earth radii)}^{3/2} / \text{min}$$

(d) Compute the drag coefficients c" and d:

$$c'' = -\frac{360 n_o^2 c_o}{\pi^2}$$

If the drag acceleration is to be used, d is read from storage or, if zero is found there, it is computed from

$$d = A\,(c'')^2 \left[1 + \frac{n_D}{3(n_D - n_o)}\right]$$

where

$n_D = 0.072\ 220\ 521$ and the

empirical coefficient A = 8 (at present) ;

if $(n_D - n_o) < 10^{-10}$, set $(n_D - n_o) = 10^{-10}$

2.2.2 Atmospheric Drag

Compute the purturbations in a, e, and p caused by the Earth's atmosphere during the time interval $(t - t_o)$. At time t:

$$a = a_o \left[1 + 2\, c''\, (t - t_o) + 3d\, (t - t_o)^2 \right]^{-2/3}$$

$$e = 1 - q_o/a \quad \text{for} \quad a \geq q_o \quad \text{and}$$

$$e = 0 \quad \text{for} \quad a < q_o$$

$$p = a(1 - e^2)$$

2.2.3 Asphericity of the Earth

Using the initial values of p_o, n_o, and i_o, calculate the secular bulge effects on Ω, ω, a_{xN} and a_{yN}.

$$\frac{d\,\Omega}{dt} = \Omega_1 + \Omega_2 + \Omega_3 + \Omega_4 + \Omega_5$$

$$\Omega_1 = -\frac{3}{2} J_2 \frac{a_e^{\,2}}{p_o^{\,2}} (\cos i_o)\, n_o$$

$$\Omega_2 = -\frac{3}{2} J_2^{\,2} \frac{a_e^{\,4}}{p_o^{\,4}} n_o \cos i_o \left[\frac{9}{4} - 3\sqrt{1 - e_o^{\,2}} - \sin^2 i_o \left(\frac{5}{2} - \frac{9}{2}\sqrt{1 - e_o^{\,2}} \right) \right]$$

$$\Omega_3 = -\frac{15}{16} J_4 \frac{a_e^4}{p_o^4} n_o \cos i_o \,(4 - 7 \sin^2 i_o)$$

$$\Omega_4 = -\frac{3}{8} J_2^2 \frac{a_e^4}{p_o^4} e_o^2 n_o \cos i_o \,(1 + \frac{5}{4} \sin^2 i_o)$$

$$\Omega_5 = \frac{45}{32} J_4 \frac{a_e^4}{p_o^4} e_o^2 n_o \cos i_o \,(4 - 7 \sin^2 i_o)$$

$$\Omega_{so} = \Omega_o + \frac{d\,\Omega}{dt} (t - t_o)$$

$$\frac{d\,\omega}{dt} = \omega_1 + \omega_2 + \omega_3 + \omega_4 + \omega_5$$

$$\omega_1 = \frac{3}{4} J_2 \frac{a_e^2}{p_o^2} (4 - 5 \sin^2 i_o)\, n_o$$

$$\omega_2 = \frac{9}{4} J_2^2 \frac{a_e^4}{p_o^4} (4 - 5 \sin^2 i_o)\, n_o \left[1 - \sqrt{1-e_o^2} - \sin^2 i_o \left(\frac{43}{48} - \frac{3}{2}\sqrt{1-e_o^2}\right)\right]$$

$$\omega_3 = -\frac{15}{32} J_4 \frac{a_e^4}{p_o^4} n_o \,(16 - 62 \sin^2 i_o + 49 \sin^4 i_o)$$

$$\omega_4 = \frac{9}{16} J_2^2 \frac{a_e^4}{p_o^4} e_o^2 \left[(4 - 5 \sin^2 i_o)\,(1 + \frac{1}{24} \sin^2 i_o) - \frac{5}{3} \cos^4 i_o \right] n_o$$

$$\omega_5 = -\frac{45}{128} J_4 \frac{a_e^4}{p_o^4} e_o^2 n_o \,(24 - 84 \sin^2 i_o + 63 \sin^4 i_o)$$

$$\omega_s = \frac{d\omega}{dt}(t - t_o)$$

$$\omega_{so} = \omega_o + \omega_s$$

$$\underline{W} \quad \begin{cases} W_x = \sin \Omega_{so} \sin i_o \\ W_y = -\cos \Omega_{so} \sin i_o \\ W_z = \cos i_o \end{cases}$$

$$a_{xN_S} = \frac{e}{e_o}(a_{xN_o} \cos \omega_s - a_{yN_o} \sin \omega_s)$$

$$a_{yN_S} = \frac{e}{e_o}(a_{xN_o} \sin \omega_s + a_{yN_o} \cos \omega_s)$$

If the quantity $\frac{A\gamma}{m} < 1.0$ $cm^2/gm.$, where m is the satellite's mass, the effect of direct solar radiation on the satellite orbit is not considered and the ephemeris calculation proceeds directly from Section 2.2.3 to Section 2.2.5.

If the quantity $\frac{A\gamma}{m} \geq 1.0$ $cm^2/gm.$, the perturbative effect of direct solar radiation on the satellite orbit is computed according to the formulation outlined in Section 2.2.4. This limit may be changed, if experimentation and usage show that the radiation should be applied to more or fewer satellites.

2.2.4 Computation of Radiation Pressure Corrections

The computation of the perturbative variations of the elements due to direct solar radiation pressure proceeds essentially as in Section 2.1. The eclipse factor, ν, is computed as previously described, except for the dot products $(\underline{P} \cdot \underline{L}_\odot)$ and $(\underline{Q} \cdot \underline{L}_\odot)$ which are reformulated in paragraphs f and g below. The equations for the corrections to the parameters e cos π, e sin π, W_x, W_y, and L have also been rewritten, as shown below, for programming efficiency. The a_i coefficients are calculated as shown in Table I using the relations $\Omega' = \frac{1}{k_e}\frac{d\Omega}{dt}$ and $\omega' = \frac{1}{k_e}\frac{d\omega}{dt}$. The b_i terms are computed as the reciprocals of the a_i coefficients with Ω_{so}, ω_{so}, and $\ell_\odot$ replacing Ω', ω', and $n_\odot$ respectively.

The perturbations in the elements are first computed at t, eliminating terms whose a_i coefficients are greater than 10^4. The perturbations are then recomputed at t_o, with the same terms eliminated.

The final perturbations in the elements are computed as follows:

$$\Delta\,(e \cos \pi) = \Delta(e \cos \pi)_t - \Delta(e \cos \pi)_{to}$$

$$\Delta\,(e \sin \pi) = \Delta(e \sin \pi)_t - \Delta(e \sin \pi)_{to}$$

$$\Delta W_x = (\Delta W_x)_t - (\Delta W_x)_{to}$$

$$\Delta W_y = (\Delta W_y)_t - (\Delta W_y)_{to}$$

$$\Delta L = \Delta L_t - \Delta L_{to}$$

In this way, the very long period terms are ignored; the moderately long period terms are treated as secular; and singularities caused by very long period terms are eliminated.

(a) Computation of $\Delta(e \cos \pi)$ (DECPI)

$$\begin{aligned}\Delta\,(e \cos \pi) = k_o \Big\{ & k_1 \big[2k_2 (-a_{11} \cos b_{11} + a_{12} \cos b_{12}) \\ & - k_3 (k_4 a_{13} \cos b_{13} + k_5 a_{14} \cos b_{14}) + k_6 \cos \ell_\bullet \big] \\ & + k_7 \big[k_8 (-a_{11} \cos b_{11} + a_{12} \cos b_{12} + a_{16} \cos b_{16} \\ & \qquad - a_{17} \cos b_{17}) \\ & + k_9 (-a_{14} \cos b_{14} - a_{18} \cos b_{18} + a_{20} \cos b_{20}) \\ & + k_{10} (-a_{13} \cos b_{13} - a_{19} \cos b_{19} + a_{21} \cos b_{21}) \\ & \qquad + k_{11} \cos \ell_\bullet \big] \Big\}\end{aligned}$$

(b) Computation of Δ (e $\sin \pi$) (DESPI)

$$\Delta (e \sin \pi) = k_o \Big\{ k_1 \left[2k_2 (-a_{11} \sin b_{11} + a_{12} \sin b_{12}) - k_3 (k_4 a_{13} \sin b_{13} + k_5 a_{14} \sin b_{14}) + k_{26} \sin \ell_\odot \right]$$
$$+ k_7 \left[k_8 (-a_{11} \sin b_{11} + a_{12} \sin b_{12} + a_{16} \sin b_{16} - a_{17} \sin b_{17}) \right.$$
$$+ k_9 (-a_{14} \sin b_{14} - a_{18} \sin b_{18} + a_{20} \sin b_{20})$$
$$+ k_{10} (-a_{13} \sin b_{13} - a_{19} \sin b_{19} + a_{21} \sin b_{21})$$
$$\left. + k_{30} \sin \ell_\odot \right] \Big\}$$

(c) Computation of ΔW_x (DELWX)

$$\Delta W_x = k_o k_{12} \left[k_{13} (-a_1 \sin b_1 + a_2 \sin b_2) \right.$$
$$+ k_{14} (a_3 \sin b_3 + a_4 \sin b_4)$$
$$+ k_{15} a_5 \sin b_5 + k_{16} a_6 \sin b_6 + k_{17} a_7 \sin b_7$$
$$\left. + k_{18} a_8 \sin b_8 - k_{19} a_9 \sin b_9 - k_{20} a_{10} \sin b_{10} \right]$$

(d) Computation of ΔW_y (DELWY)

$$\Delta W_y = k_o k_{12} \left[k_{13} (a_1 \cos b_1 - a_2 \cos b_2) \right.$$
$$+ k_{14} (a_3 \cos b_3 + a_4 \cos b_4)$$
$$+ k_{28} a_5 \cos b_5 + k_{29} a_6 \cos b_6 - k_{17} a_7 \cos b_7$$
$$\left. - k_{18} a_8 \cos b_8 + k_{19} a_9 \cos b_9 + k_{20} a_{10} \cos b_{10} \right]$$

(e) Computation of ΔL (DLTL)

$$L = k_o \Big\{ k_{21} \, [k_{22} \, a_1 \sin b_1 + k_{23} \, a_2 \sin b_2 + k_{24} \, a_3 \sin b_3 + k_{25} \, a_4 \sin b_4 + k_2 \, (-a_5 \sin b_5 + a_6 \sin b_6)] + k_{27} \, [k_{10} \, (-a_1 \sin b_1 + a_3 \sin b_3) + k_9 \, (a_4 \sin b_4 - a_2 \sin b_2) + k_8 \, (-a_5 \sin b_5 + a_6 \sin b_6)] \Big\}$$

(f) Computation of $(\underline{P} \cdot \underline{L}_\odot)$ (PDØTL)

$$4(\underline{P} \cdot \underline{L}_\odot) = k_{25} \cos\left(b_4^{-1}\right) + k_{24} \cos\left(b_3^{-1}\right) + k_{23} \cos\left(b_2^{-1}\right) + k_{22} \cos\left(b_1^{-1}\right) + k_2 \left[\cos\left(b_6^{-1}\right) - \cos\left(b_5^{-1}\right) \right]$$

(g) Computation of $(\underline{Q} \cdot \underline{L}_\odot)$ (QDØTL)

$$4(\underline{Q} \cdot \underline{L}_\odot) = k_{25} \sin\left(b_4^{-1}\right) + k_{24} \sin\left(b_3^{-1}\right) - k_{23} \sin\left(b_2^{-1}\right) - k_{22} \sin\left(b_1^{-1}\right) + k_2 \left[- \sin\left(b_6^{-1}\right) + \sin\left(b_5^{-1}\right) \right]$$

TABLE II

The Coefficients k_0 - k_{30}

Coefficient	Symbol		Formula
k_0	XYZK0	=	$-\frac{3}{16}$ [PSUN·AGØM·XYZNU]
k_1	XYZK1	=	RTAUP·RTEØSQ
k_2	XYZK2	=	2·SINI·SINEP
k_3	XYZK3	=	2(1 - CØSI)
k_4	XYZK4	=	(1 - CØSEP)
k_5	XYZK5	=	(1 + CØSEP)
k_6	XYZK6	=	$\frac{-4}{\text{XNSUN}}$ [(1 + CØSI) (CØSEP)]
k_7	XYZK7	=	$\frac{\text{RTAUP } (E\phi)^2 \text{ SINI}}{\text{RTEØSQ } (1 + \text{CØSI})}$
k_8	XYZK8	=	2 CØSI·SINEP
k_9	XYZK9	=	SINI (1 + CØSEP)
k_{10}	XYZK10	=	SINI (1 - CØSEP)
k_{11}	XYZK11	=	$\frac{2}{\text{XNSUN}}$ (SINI) (CØSEP)
k_{12}	XYZK12	=	$-\frac{\text{RTAUP}}{\text{RTEØSQ}} \cdot E\phi$
k_{13}	XYZK13	=	2 CØSI SINEP (1 + CØSI)
k_{14}	XYZK14	=	2 CØSI SINEP (1 - CØSI)
k_{15}	XYZK15	=	2 SINI (CØSI + CØSEP)
k_{16}	XYZK16	=	2 SINI (CØSI - CØSEP)

TABLE II (Continued)

Coefficient	Symbol		Formula
k_{17}	XYZK17	=	SINI (1 - CØSI) (1 + CØSEP)
k_{18}	XYZK18	=	SINI (1 - CØSI) (1 - CØSEP)
k_{19}	XYZK19	=	SINI (1 + CØSI) (1 + CØSEP)
k_{20}	XYZK20	=	SINI (1 + CØSI) (1 - CØSEP)
k_{21}	XYZK21	=	$4\ E\phi \cdot \mathrm{RTAUP} \left[1 - \frac{\mathrm{RTE\emptyset SQ}}{2(\mathrm{RTE\emptyset SQ}+1)}\right]$
k_{22}	XYZK22	=	(1 + CØSI) (1 - CØSEP)
k_{23}	XYZK23	=	(1 + CØSI) (1 + CØSEP)
k_{24}	XYZK24	=	(1 - CØSI) (1 - CØSEP)
k_{25}	XYZK25	=	(1 - CØSI) (1 + CØSEP)
k_{26}	XYZK26	=	$\frac{-4}{\mathrm{XNSUN}}$ (1 + CØSI)
k_{27}	XYZK27	=	$-\frac{2E\phi \cdot \mathrm{RTAUP} \cdot \mathrm{SINI}}{\mathrm{RTE\emptyset SQ}(1 + \mathrm{C\emptyset SI})}$
k_{28}	XYZK28	=	-2SINI (1 + CØSI·CØSEP)
k_{29}	XYZK29	=	-2SINI (1 - CØSI·CØSEP)
k_{30}	XYZK30	=	$\frac{2\mathrm{SINI}}{\mathrm{XNSUN}}$

(h) Application of Perturbations in the Elements

The corrective quantities Δa, Δ (e cos π), Δ (e sin π), ΔW_x, ΔW_y, and ΔL due to direct solar radiation are now applied to the orbital elements at time t. The perturbative effects of the Earth's asphericity and atmosphere have previously been included in the elements a, $a_{xN} = e \cos \omega$, $a_{yN} = e \sin \omega$, Ω, and L. Since the radiation pressure perturbations are based instead on the parameters (e cos π), (e sin π), W_x, and W_y, they must be introduced in the following manner.

(1) Calculate (e cos π) and (e sin π) at time t:

$$e \cos \pi = [a_{xN} \cos \Omega - a_{yN} \sin \Omega] + \Delta (e \cos \pi)$$

$$e \sin \pi = [a_{xN} \sin \Omega + a_{yN} \cos \Omega] + \Delta (e \sin \pi)$$

The square-bracketed terms include the bulge and drag perturbations at time t.

(2) The eccentricity, which now includes the perturbative effects of the bulge, atmospheric drag, and the radiation pressure, is computed from

$$e = [(e \cos \pi)^2 + (e \sin \pi)^2]^{1/2}$$

(3) Similarly, $\underline{W}$ is computed from

$$W_x = [W_x] + \Delta W_x$$

$$W_y = [W_y] + \Delta W_y$$

$$W_z = [W_z] - \left(\frac{W_x \Delta W_x + W_y \Delta W_y}{\cos i} \right)$$

where the square-bracketed components of $\underline{W}$, computed in 2.2.3 above, include the bulge perturbations. If $|\cos i|$ is too small, compute

$$|W_z| = \sqrt{1 - W_x^2 - W_y^2} \text{ and apply the sign of } [-W_x \Delta W_x - W_y \Delta W_y].$$

(4) The inclination i and Ω are now revised, both radiation pressure and secular bulge effects being included:

$$\cos i = W_z \quad , \quad \sin i = \sqrt{1 - \cos^2 i}$$

$$i = \tan^{-1} \left[\frac{\sin i}{\cos i}\right] ; 0 < i < 180^o$$

$$\cos \Omega = \frac{- W_y}{\sin i} \quad , \quad \sin \Omega = \frac{W_x}{\sin i}$$

$$\Omega = \tan^{-1} \left[\frac{\sin \Omega}{\cos \Omega}\right] ; \; 0 < \Omega < 2 \pi$$

(5) Recompute a_{xN}, a_{yN} with the new Ω :

$$a_{xN} = (e \cos \pi) \cos \Omega + (e \sin \pi) \sin \Omega$$

$$a_{yN} = - (e \cos \pi) \sin \Omega + (e \sin \pi) \cos \Omega$$

$(e \cos \pi)$ and $(e \sin \pi)$ are given by 2.2.4 (h), (1).

2.2.5 Geocentric Position and Velocity Calculations

(a) Calculate the long-period effects on Ω , a_{xN}, and a_{yN}:

$$\Omega_L = \Omega_6 + \Omega_7 + \Omega_8 + \Omega_9 + \Omega_{10}$$

$$* \Omega_6 = - \frac{1}{8} J_2 \frac{a_e^2}{p^2} e^2 \cos i_o \frac{(7-15 \sin^2 i_o)}{(4-5 \sin^2 i_o)} \sin 2 \omega_{so}$$

$$** \Omega_7 = - \frac{5}{16} J_2 \frac{a_e^2}{p^2} e^2 \sin^2 i_o \cos i_o \frac{(14-15 \sin^2 i_o)}{(4-5 \sin^2 i_o)^2} \sin 2 \omega_{so}$$

$$\Omega_8 = - \frac{1}{2} \frac{J_3}{J_2} \frac{a_e}{p} e \frac{\cos i_o}{\sin i_o} \cos \omega_{so}$$

Note: All asterisks denote program size tests explained at the end of this Section, 2.2.5

where

$$\frac{1}{2}\ \frac{J_3}{J_2}\ a_e = -\ 1.1548 \times 10^{-3}$$

$$*\ \Omega_9 = -\ \frac{5}{8}\ \frac{J_4}{J_2}\ \frac{a_e^{\ 2}}{p^2}\ e^2 \cos i_o\ \frac{(3-7 \sin^2 i_o)}{(4-5 \sin^2 i_o)} \sin 2\ \omega_{so}$$

where

$$\frac{5}{8}\ \frac{J_4}{J_2}\ a_e^{\ 2} = -\ 1.0682 \times 10^{-3}$$

$$**\ \Omega_{10} = -\ \frac{25}{16}\ \frac{J_4}{J_2}\ \frac{a_e}{p^2}\ e^2 \cos i_o\ \sin^2 i_o\ \frac{(6-7 \sin^2 i_o)}{(4-5 \sin^2 i_o)} \sin 2\ \omega_{so}$$

$$\Omega_{sL} = \Omega_{so} + \Omega_L$$

$$a_{xN_L} = a_{xN_1} + a_{xN_2} + a_{xN_3} + a_{xN_4} + a_{xN_5} + a_{xN_6} + a_{xN_7} + a_{xN_8} + a_{xN_9} + a_{xN_{10}} + a_{xN_{11}}$$

$$*\ a_{xN_1} = \frac{1}{16}\ J_2\ \frac{a_e^{\ 2}}{p^2}\ e \sin^2 i_o\ \frac{(14-15 \sin^2 i_o)}{(4-5 \sin^2 i_o)}\ \cos \omega_{so}$$

$$*\ a_{xN_2} = -\ \frac{1}{32}\ \frac{a_e^{\ 2}}{p^2}\ e^3\ \frac{(13-15 \sin^2 i_o + 105/2 \sin^4 i_o)}{(4-5 \sin^2 i_o)}\ \cos \omega_{so}$$

Note: All asterisks denote program size tests explained at the end of this Section, 2.2.5

** $$a_{xN_3} = -\frac{1}{32} J_2 \frac{a_e^2}{p^2} e^3 \sin^2 i_o \frac{(13-15 \sin^2 i_o)(14-15 \sin^2 i_o)}{(4-5 \sin^2 i_o)^2} \cos \omega_{so}$$

* $$a_{xN_4} = \frac{1}{32} J_2 \frac{a_e^2}{p^2} e^3 \frac{(14-93 \sin^2 i_o + \frac{165}{2} \sin^4 i_o)}{(4-5 \sin^2 i_o)} \cos 3\omega_{so}$$

** $$a_{xN_5} = \frac{1}{32} J_2 \frac{a_e^2}{p^2} e^3 \sin^2 i_o \frac{(13-15 \sin^2 i_o)(14-15 \sin^2 i_o)}{(4-5 \sin^2 i_o)^2} \cos 3\omega_{so}$$

$$a_{xN_6} = -\frac{1}{4} \frac{J_3}{J_2} \frac{a_e}{p} e^2 \sin^2 i_o \frac{(1-2 \sin^2 i_o)}{\sin i_o} \sin 2\omega_{so}$$

* $$a_{xN_7} = \frac{5}{16} \frac{J_4}{J_2} \frac{a_e^2}{p^2} e \sin^2 i_o \frac{(6-7 \sin^2 i_o)}{(4-5 \sin^2 i_o)} \cos \omega_{so}$$

* $$a_{xN_8} = -\frac{5}{32} \frac{J_4}{J_2} \frac{a_e^2}{p^2} e^3 \frac{(6-29 \sin^2 i_o + 49/2 \sin^4 i_o)}{(4-5 \sin^2 i_o)} \cos \omega_{so}$$

** $$a_{xN_9} = -\frac{5}{32} \frac{J_4}{J_2} \frac{a_e^2}{p^2} e^3 \sin^2 i_o \frac{(6-7 \sin^2 i_o)(13-15 \sin^2 i_o)}{(4-5 \sin^2 i_o)^2} \cos \omega_{so}$$

* $$a_{xN_{10}} = \frac{5}{32} \frac{J_4}{J_2} \frac{a_e^2}{p^2} e^3 \frac{(6-41 \sin^2 i_o + \frac{77}{2} \sin^4 i_o)}{(4-5 \sin^2 i_o)} \cos 3\omega_{so}$$

Note: All asterisks denote program size tests explained at the end of this Section, 2.2.5

$$** \quad a_{xN_{11}} = \frac{5}{32} \frac{J_4}{J_2} \frac{a_e^2}{p^2} e^3 \sin^2 i_o \frac{(6-7 \sin^2 i_o)(13-15 \sin^2 i_o)}{(4-5 \sin^2 i_o)^2} \cos 3\,\omega_{so}$$

$$a_{yN_L} = a_{yN_1} + a_{yN_2} + a_{yN_3} + a_{yN_4} + a_{yN_5} + a_{yN_6} + a_{yN_7} + a_{yN_8} + a_{yN_9}$$

$$+ a_{yN_{10}} + a_{yN_{11}} + a_{yN_{12}} + a_{yN_{13}}$$

$$* \quad a_{yN_1} = -\frac{1}{16} J_2 \frac{a_e^2}{p^2} e \sin^2 i_o \frac{(14-15 \sin^2 i_o)}{(4-5 \sin^2 i_o)} \sin \omega_{so}$$

$$* \quad a_{yN_2} = \frac{1}{32} J_2 \frac{a_e^2}{p^2} e^3 \frac{(14-65 \sin^2 i_o + \frac{105}{2} \sin^4 i_o)}{(4-5 \sin^2 i_o)} \sin \omega_{so}$$

$$** \quad a_{yN_3} = \frac{1}{32} J_2 \frac{a_e^2}{p^2} e^3 \sin^2 i_o \frac{(13-15 \sin^2 i_o)(14-15 \sin^2 i_o)}{(4-5 \sin^2 i_o)^2} \sin \omega_{so}$$

$$* \quad a_{yN_4} = \frac{1}{32} J_2 \frac{a_e^2}{p^2} e^3 \frac{(14-93 \sin^2 i_o + \frac{165}{2} \sin^4 i_o)}{(4-5 \sin^2 i_o)} \sin 3\,\omega_{so}$$

$$** \quad a_{yN_5} = \frac{1}{32} J_2 \frac{a_e^2}{p^2} e^3 \sin^2 i_o \frac{(13-15 \sin^2 i_o)(14-15 \sin^2 i_o)}{(4-5 \sin^2 i_o)^2} \sin 3\omega_{so}$$

$$a_{yN_6} = -\frac{1}{2} \frac{J_3}{J_2} \frac{a_e}{p} \sin i_o$$

Note: All asterisks denote program size tests explained at the end of this Section, 2.2.5

$$a_{yN_7} = \frac{1}{4} \frac{J_3}{J_2} \frac{a_e}{p} \frac{e^2}{\sin i_o}$$

$$a_{yN_8} = \frac{1}{4} \frac{J_3}{J_2} \frac{a_e}{p} e^2 \frac{(1-2 \sin^2 i_o)}{\sin i_o} \cos 2 \omega_{so}$$

$$* \quad a_{yN_9} = -\frac{5}{16} \frac{J_4}{J_2} \frac{a_e^2}{p^2} e \sin^2 i_o \frac{(6-7 \sin^2 i_o)}{(4-5 \sin^2 i_o)} \sin \omega_{so}$$

$$* \quad a_{yN_{10}} = \frac{5}{32} \frac{J_4}{J_2} \frac{a_e^2}{p^2} e^3 \frac{(6-29 \sin^2 i_o + \frac{49}{2} \sin^4 i_o)}{(4-5 \sin^2 i_o)} \sin \omega_{so}$$

$$** \quad a_{yN_{11}} = \frac{5}{32} \frac{J_4}{J_2} \frac{a_e^2}{p^2} e^3 \sin^2 i_o \frac{(6-7 \sin^2 i_o)(13-15 \sin^2 i_o)}{(4-5 \sin^2 i_o)^2} \sin \omega_{so}$$

$$* \quad a_{yN_{12}} = \frac{5}{32} \frac{J_4}{J_2} \frac{a_e^2}{p^2} e^3 \frac{(6-41 \sin^2 i_o + \frac{77}{2} \sin^4 i_o)}{(4-5 \sin^2 i_o)} \sin 3 \omega_{so}$$

$$** \quad a_{yN_{13}} = \frac{5}{32} \frac{J_4}{J_2} \frac{a_e^2}{p^2} e^3 \sin^2 i_o \frac{(6-7 \sin^2 i_o)(13-15 \sin^2 i_o)}{(4-5 \sin^2 i_o)^2} \sin 3 \omega_{so}$$

$$a_{xN_{SL}} = a_{xN_S} + a_{xN_L}$$

Note: All asterisks denote program size tests explained at the end of this Section, 2.2.5

$$ {}^{a}yN_{SL} = {}^{a}yN_{S} + {}^{a}yN_{L} $$

(b) Compute the mean longitude L at time t:

$$ L = L_o + n_o\,(1+\Delta M+\Delta\pi)\,(t-t_o) + n_o c''\,(t-t_o)^2 + n_o d\,(t-t_o)^3 + L_L + \Delta L $$

where $n_o \Delta M$, $n_o \Delta\pi$, and L_L are computed as in the following three subsections, and ΔL is the effect of solar radiation pressure (Section 2.2.4) or is set equal to zero if radiation pressure effects are not considered.

(1) First and second-order secular changes in the mean anomaly M:

$$ n_o \Delta M = M_1 + M_2 + M_3 $$

$$ M_1 = \frac{3}{2}\, J_2^2\, \frac{a_e^4}{p_o^4}\, \sqrt{1-e_o^2}\, \left[-\frac{25}{32} + \frac{5}{2}\sqrt{1-e_o^2} + \left(\frac{15}{16} - 3\sqrt{1-e_o^2}\right) \sin^2 i_o + \left(\frac{65}{32} - \frac{9}{2}\sqrt{1-e_o^2}\right) \cos^4 i_o \right] n_o $$

$$ M_2 = \frac{15}{64}\, J_2^2\, \frac{a_e^4}{p_o^4}\, \sqrt{1-e_o^2}\;\; e_o^2\, \left(\frac{13}{2} - 9 \sin^2 i_o - \frac{5}{2} \cos^4 i_o\right) n_o $$

$$ M_3 = \frac{45}{128}\, J_4\, \frac{a_e^4}{p_o^4}\, \sqrt{1-e_o^2}\;\; e_o^2\, (27 - 30 \sin^2 i_o - 35 \cos^4 i_o)\, n_o $$

(2) First and second-order secular changes in the argument of perigee π :

$$ n_o \Delta\pi = \pi_1 + \pi_2 + \pi_3 + \pi_4 + \pi_5 $$

$$\pi_1 = \frac{3}{2} \; J_2 \; \frac{a_e^{\;2}}{p_o^{\;2}} \; \left(2 - \frac{5}{2}\sin^2 i_o - \left|\cos i_o\right| \right) n_o$$

$$\pi_2 = \frac{9}{4} \; J_2^{\;2} \; \frac{a_e^{\;4}}{p_o^{\;4}} \left\{ (4-5\sin^2 i_o) \left[1 - \sqrt{1-e_o^{\;2}} - \left(\frac{43}{48} - \frac{3}{2}\sqrt{1-e_o^{\;2}}\right)\sin^2 i_o \right] \right.$$

$$\left. - \left|\cos i_o\right| \left[\frac{3}{2} - 2\sqrt{1-e_o^{\;2}} - \sin^2 i_o \left(\frac{5}{3} - 3\sqrt{1-e_o^{\;2}}\right) \right] \right\} n_o$$

$$\pi_3 = \frac{9}{16} \; J_2^{\;2} \; \frac{a_e^{\;4}}{p_o^{\;4}} \; e_o^{\;2} \left\{ (4-5\sin^2 i_o)\left(1 + \frac{1}{24}\sin^2 i_o\right) - \frac{5}{3}\cos^4 i_o \right.$$

$$\left. - \frac{1}{6}\left|\cos i_o\right| \; (4+5\sin^2 i_o) \right\} n_o$$

$$\pi_4 = -\frac{15}{32} \; J_4 \; \frac{a_e^{\;4}}{p_o^{\;4}} \left[16 - 62\sin^2 i_o + 49\sin^4 i_o \right.$$

$$\left. -2\left|\cos i_o\right| (4-7\sin^2 i_o) \right] n_o$$

$$\pi_5 = -\frac{15}{128} \; J_4 \; \frac{a_e^{\;4}}{p_o^{\;4}} \; e_o^{\;2} \left[72 - 252\sin^2 i_o + 189\sin^4 i_o \right.$$

$$\left. -12\left|\cos i_o\right| (4-7\sin^2 i_o) \right] n_o$$

(3) Long-period changes in L:

$$L_L = L_1 + L_2 + L_3 + L_4 + L_5 + L_6$$

$$* \quad L_1 = \frac{1}{16} J_2 \frac{a_e^{\ 2}}{p^2 (4-5 \sin^2 i_o)} \Big\{ [(1-e^2)^{3/2} - 1] (14-15 \sin^2 i_o) \sin^2 i_o$$

$$+ e^2 \left[14-79 \sin^2 i_o + \frac{135}{2} \sin^4 i_o - |\cos i_o| (14-30 \sin^2 i_o) \right] \Big\} \sin 2\, \omega_{so}$$

$$** \quad L_2 = \frac{1}{16} J_2 \frac{a_e^{\ 2}}{p^2} e^2 \frac{\sin^2 i_o (14-15 \sin^2 i_o)}{(4-5 \sin^2 i_o)^2} [(13-15 \sin^2 i_o)$$

$$- 5 |\cos i_o|] \sin 2\, \omega_{so}$$

$$\dagger L_3 = \frac{1}{2} \frac{J_3}{J_2} \frac{a_e}{p} \left[\frac{(1-e^2)^{3/2} - 1}{e} \sin i_o \right] \cos \omega_{so}$$

$$L_4 = \frac{1}{2} \frac{J_3}{J_2} \frac{a_e}{p} e \frac{|\cos i_o|}{\sin i_o} (|\cos i_o| - 1) \cos \omega_{so}$$

$$* \quad L_5 = \frac{5}{16} \frac{J_4}{J_2} \frac{a_e^{\ 2}}{p^2 (4-5 \sin^2 i_o)} \Big\{ [(1-e^2)^{3/2} - 1] (6-7 \sin^2 i_o) \sin^2 i_o$$

$$+ e^2 \left[6-35 \sin^2 i_o + \frac{63}{2} \sin^4 i_o - |\cos i_o| (6-14 \sin^2 i_o) \right] \Big\} \sin 2\, \omega_{so}$$

Note: All asterisks and daggers denote program size tests explained at the end of this Section, 2.2.5

** $L_6 = \frac{5}{16} \frac{J_4}{J_2} \frac{a_e^2}{p^2} e^2 \frac{\sin^2 i_o (6-7 \sin^2 i_o)}{(4-5 \sin^2 i_o)^2} \left[(13-15 \sin^2 i_o) \right.$

$\left. -5 \left| \cos i_o \right| \right] \sin 2\omega_{so}$

(c) Compute the mean argument of latitude, U, at time t:

$U = L - \Omega_{sL}$ if $W_z \geq 0$

$U = L + \Omega_{sL}$ if $W_z < 0$

(d) Solve the following modified form of Kepler's equation for the quantity $(E+\omega)$ by iteration, using U (mod 2π) as a first guess

$$E + \omega = U + a_{xN_{SL}} \sin (E+\omega) - a_{yN_{SL}} \cos (E+\omega)$$

(e) Compute the geocentric position, $\underline{r}$, and velocity, $\dot{\underline{r}}$, at time t by means of the following sequence of equations:

$$e \cos E = a_{xN_{SL}} \cos (E+\omega) + a_{yN_{SL}} \sin (E+\omega)$$

$$e \sin E = a_{xN_{SL}} \sin (E+\omega) - a_{yN_{SL}} \cos (E+\omega)$$

$$e_L^2 = a_{xN_{SL}}^2 + a_{yN_{SL}}^2$$

$$r = a (1-e \cos E)$$

$$\dot{r} = \frac{\sqrt{\mu a}}{r} e \sin E; \quad \mu = 1$$

Note: All asterisks denote program size tests explained at the end of this Section, 2.2.5

$$r\dot{v} = \sqrt{\frac{\mu a}{r}}\sqrt{1-e_L^2}$$

$$\cos u = \frac{a}{r}\left[\cos(E+\omega) - a_{xN_{SL}} + a_{yN_{SL}}\left(\frac{e \sin E}{1+\sqrt{1-e_L^2}}\right)\right]$$

$$\sin u = \frac{a}{r}\left[\sin(E+\omega) - a_{yN_{SL}} - a_{xN_{SL}}\left(\frac{e \sin E}{1+\sqrt{1-e_L^2}}\right)\right]$$

$$p_L = a\,(1-e_L^2)$$

(1) Long-period terms in i:

$$i_{oL} = i_o + i_L$$

$$i_L = i_1 + i_2 + i_3$$

$$* \; i_1 = -\frac{1}{32} J_2 \frac{a_e^2}{p^2} e^2 \sin 2i_o \frac{(14-15 \sin^2 i_o)}{(4-5 \sin^2 i_o)} \cos 2\omega_{so}$$

$$i_2 = \frac{1}{2} \frac{J_3}{J_2} \frac{a_e}{p} e \cos i_o \sin \omega_{so}$$

$$* \; i_3 = -\frac{5}{32} \frac{J_4}{J_2} \frac{a_e^2}{p^2} e^2 \sin 2i_o \frac{(6-7 \sin^2 i_o)}{(4-5 \sin^2 i_o)} \cos 2\omega_{so}$$

(2) Short-period terms in argument of latitude u:

$$\Delta u = u_1 + u_2 + u_3 + u_4 + u_5 + u_6$$

$$u_1 = -\frac{1}{8} J_2 \frac{a_e^2}{p_L^2} (6-7 \sin^2 i_{oL}) \sin 2u$$

Note: All asterisks denote program size tests explained at the end of this Section, 2.2.5

$$u_2 = \frac{3}{4} J_2 \frac{a_e^2}{p_L^2} (4-5 \sin^2 i_{oL}) (u-U + a_{xN_{SL}} \sin u - a_{yN_{SL}} \cos u)$$

$$\dagger\dagger\ u_3 = J_2 \frac{a_e^2}{p_L^2} (1- \frac{3}{2} \sin^2 i_{oL}) \left(\frac{1- \sqrt{1-e_L^2}}{e_L^2} - \frac{1}{2}\right) (a_{xN_{SL}} \sin u - a_{yN_{SL}} \cos u)$$

$$\dagger\dagger\ u_4 = \frac{1}{4} J_2 \frac{a_e^2}{p_L^2} (1 - \frac{3}{2} \sin^2 i_{oL}) \left(\frac{1- \sqrt{1-e_L^2}}{e_L^2}\right) \left[(a_{xN_{SL}}^2 - a_{yN_{SL}}^2) \sin 2u - 2\, a_{xN_{SL}}\, a_{yN_{SL}} \cos 2u \right]$$

$$u_5 = - \frac{1}{4} J_2 \frac{a_e^2}{p_L^2} (3-5 \sin^2 i_{oL}) (a_{xN_{SL}} \sin u + a_{yN_{SL}} \cos u)$$

$$u_6 = - \frac{1}{4} J_2 \frac{a_e^2}{p_L^2} \cos^2 i_{oL} (a_{xN_{SL}} \sin 3u - a_{yN_{SL}} \cos 3u)$$

(3) Short-period terms in radial distance r:

$$\Delta r = r_1 + r_2 + r_3$$

$$r_1 = - \frac{1}{2} J_2 \frac{a_e^2}{p_L^2} (1 - \frac{3}{2} \sin^2 i_{oL}) (p_L - \sqrt{1-e_L^2}\, r)$$

Note: The daggers also indicate program tests explained at the end of this Section, 2.2.5.

$$r_2 = \frac{1}{4} J_2 \frac{a_e^2}{p_L} \sin^2 i_{oL} \cos 2u$$

$$\dagger\dagger r_3 = -\frac{1}{2} J_2 \frac{a_e^2}{p_L} \left(1 - \frac{3}{2} \sin^2 i_{oL}\right) \frac{\left(1 - \sqrt{1-e_L^2}\right)}{e_L^2} \left(\sqrt{\frac{p_L}{r}} - 1\right)$$

(4) Short-period terms in $\dot{r}$:

$$\Delta \dot{r} = \dot{r}_1 + \dot{r}_2 + \dot{r}_3 + \dot{r}_4 + \dot{r}_5 + \dot{r}_6 + \dot{r}_7 + \dot{r}_8 + \dot{r}_9$$

$$\dagger\dagger\dagger \dot{r}_1 = \frac{1}{2} J_2 \frac{a_e^2}{p_L^2} \sqrt{\frac{\mu}{p_L}} \left(1 - \frac{3}{2} \sin^2 i_{oL}\right) \left[\frac{1 - \left(1-e_L^2\right)^{3/2}}{e_L^2} + \frac{1}{4}\left(1 - 7\sqrt{1-e_L^2}\right)\right] \left(a_{xN_{SL}} \sin u - a_{yN_{SL}} \cos u\right)$$

$$\dagger\dagger \dot{r}_2 = \frac{1}{2} J_2 \frac{a_e^2}{p_L^2} \sqrt{\frac{\mu}{p_L}} \left(1 - \frac{3}{2} \sin^2 i_{oL}\right) \left(\frac{1 - \sqrt{1-e_L^2}}{e_L^2}\right) \times \left[\left(a_{xN_{SL}}^2 - a_{yN_{SL}}^2\right) \sin 2u - 2\, a_{xN_{SL}} a_{yN_{SL}} \cos 2u\right]$$

Note: All daggers denote program size tests explained at the end of this Section, 2.2.5

$$\dagger\dagger\ \dot{r}_3 = +\frac{1}{8}\ J_2\ \frac{a_e^2}{p_L^2}\sqrt{\frac{\mu}{p_L}}\ \frac{1-\sqrt{1-e_L^2}}{e_L^2}\left(1-\frac{3}{2}\sin^2 i_{oL}\right)\ (a_{xN_{SL}}\sin u - a_{yN_{SL}}\cos u)$$

$$\times\left[3e^2 - 4(a_{xN_{SL}}\sin u - a_{yN_{SL}}\cos u)^2\right]$$

$$\dot{r}_4 = -\frac{1}{4}\ J_2\ \frac{a_e^2}{p_L^2}\sqrt{\frac{\mu}{p_L}}\ \sin^2 i_{oL}\, a_{xN_{SL}}\, a_{yN_{SL}}$$

$$\dot{r}_5 = -\frac{1}{2}\ J_2\ \frac{a_e^2}{p_L^2}\sqrt{\frac{\mu}{p_L}}\ \sin^2 i_{oL}(a_{xN_{SL}}\sin u + a_{yN_{SL}}\cos u)$$

$$\dot{r}_6 = -\frac{1}{2}\ J_2\ \frac{a_e^2}{p_L^2}\sqrt{\frac{\mu}{p_L}}\ \sin^2 i_{oL}\sin 2u$$

$$\dot{r}_7 = -\frac{3}{8}\ J_2\ \frac{a_e^2}{p_L^2}\sqrt{\frac{\mu}{p_L}}\ e_L^2\ \sin^2 i_{oL}\sin 2u$$

$$\dot{r}_8 = -\frac{1}{2}\ J_2\ \frac{a_e^2}{p_L^2}\sqrt{\frac{\mu}{p_L}}\ \sin^2 i_{oL}(a_{xN_{SL}}\sin 3u - a_{yN_{SL}}\cos 3u)$$

$$\dot{r}_9 = -\frac{1}{8}\ J_2\ \frac{a_e^2}{p_L^2}\sqrt{\frac{\mu}{p_L}}\ \sin^2 i_{oL}\left[(a_{xN_{SL}}^2 - a_{yN_{SL}}^2)\sin 4u - 2a_{xN_{SL}}a_{yN_{SL}}\cos 4u\right]$$

(5) Short-period terms in $r\dot{v}$ ($r^2\dot{v} = \sqrt{\mu p}$):

$$(r\dot{v}) = r\dot{v}_1 + r\dot{v}_2 + r\dot{v}_3 + r\dot{v}_4 + r\dot{v}_5 + r\dot{v}_6 + r\dot{v}_7 + r\dot{v}_8$$

$$r\dot{v}_1 = -\frac{r\dot{v}}{r}\ \Delta r$$

Note: All daggers denote program size tests expla..ned at the end of this Section, 2.2.5

$$r\dot{v}_2 = \frac{3}{4} J_2 \frac{a_e^2}{p_L^2} \sqrt{\frac{\mu}{p_L}} \sqrt{1-e_L^2} \left(1- \frac{3}{2} \sin^2 i_{oL}\right) \left(\frac{p_L}{r}\right)$$

$$r\dot{v}_3 = \frac{3}{4} J_2 \frac{a_e^2}{p_L^2} \sqrt{\frac{\mu}{p_L}} \sin^2 i_{oL} \cos 2u$$

$$r\dot{v}_4 = \frac{1}{2} J_2 \frac{a_e^2}{p_L^2} \sqrt{\frac{\mu}{p_L}} e_L^2 \sin^2 i_{oL} \cos 2u$$

$$r\dot{v}_5 = \frac{9}{8} J_2 \frac{a_e^2}{p_L^2} \sqrt{\frac{\mu}{p_L}} \sin^2 i_{oL} \left(a_{xN_{SL}} \cos u - a_{yN_{SL}} \sin u\right)$$

$$r\dot{v}_6 = \frac{5}{8} J_2 \frac{a_e^2}{p_L^2} \sqrt{\frac{\mu}{p_L}} \sin^2 i_{oL} \left(a_{xN_{SL}} \cos 3u + a_{yN_{SL}} \sin 3u\right)$$

$$r\dot{v}_7 = \frac{3}{8} J_2 \frac{a_e^2}{p_L^2} \sqrt{\frac{\mu}{p_L}} \sin^2 i_{oL} \left(a^2_{xN_{SL}} - a^2_{yN_{SL}}\right)$$

$$r\dot{v}_8 = \frac{1}{8} J_2 \frac{a_e^2}{p_L^2} \sqrt{\frac{\mu}{p_L}} \sin^2 i_{oL} \left[\left(a^2_{xN_{SL}} - a^2_{yN_{SL}}\right) \cos 4u + 2a_{xN_{SL}} a_{yN_{SL}} \sin 4u\right]$$

(6) Short-period terms in Ω:

$$\Delta\Omega = \Omega_{11} + \Omega_{12} + \Omega_{13} + \Omega_{14}$$

$$\Omega_{11} = \frac{3}{4} J_2 \frac{a_e^2}{p_L^2} \cos i_{oL} \sin 2u$$

$$\Omega_{12} = -\frac{3}{2} J_2 \frac{a_e^2}{p_L^2} \cos i_{oL} (u - U + a_{xN_{SL}} \sin u - a_{yN_{SL}} \cos u)$$

$$\Omega_{13} = \frac{3}{4} J_2 \frac{a_e^2}{p_L^2} \cos i_{oL} (a_{xN_{SL}} \sin u + a_{yN_{SL}} \cos u)$$

$$\Omega_{14} = \frac{1}{4} J_2 \frac{a_e^2}{p_L^2} \cos i_{oL} (a_{xN_{SL}} \sin 3u - a_{yN_{SL}} \cos 3u)$$

(7) Short-period terms in i:

$$\Delta i = i_4 + i_5 + i_6$$

$$i_4 = \frac{3}{8} J_2 \frac{a_e^2}{p_L^2} \sin 2i_{oL} \cos 2u$$

$$i_5 = \frac{3}{8} J_2 \frac{a_e^2}{p_L^2} \sin 2i_{oL} (a_{xN_{SL}} \cos u - a_{yN_{SL}} \sin u)$$

$$i_6 = \frac{1}{8} J_2 \frac{a_e^2}{p_L^2} \sin 2i_{oL} (a_{xN_{SL}} \cos 3u + a_{yN_{SL}} \sin 3u)$$

(8) Orientation vectors $\underline{W}$, $\underline{M}$, $\underline{N}$:

$$i_k = i_{oL} + \Delta i$$

$$\Omega_k = \Omega_{sL} + \Delta\Omega$$

$$\left.\begin{aligned} W_x &= \sin \Omega_k \sin i_k \\ W_y &= - \cos \Omega_k \sin i_k \\ W_z &= \cos i_k \end{aligned}\right\} \quad \underline{W}$$

$$\left.\begin{aligned} M_x &= \sin \Omega_k \cos i_k \\ M_y &= \cos \Omega_k \cos i_k \\ M_z &= \sin i_k \end{aligned}\right\} \quad \underline{M}$$

$$\left.\begin{aligned} N_x &= \cos \Omega_k \\ N_y &= \sin \Omega_k \\ N_z &= 0 \end{aligned}\right\} \quad \underline{N}$$

(9) Direction vectors $\underline{U}$, $\underline{V}$:

$$u_k = u + \Delta u$$

$$\underline{U} = \underline{N} \cos u_k + \underline{M} \sin u_k$$

$$\underline{V} = -\underline{N} \sin u_k + \underline{M} \cos u_k$$

(10) **Position and velocity $\underline{r}$ and $\underline{\dot{r}}$:**

$$r_k = r + \Delta r$$

$$\dot{r}_k = \dot{r} + \Delta \dot{r}$$

$$(r\dot{v})_k = r\dot{v} + \Delta (r\dot{v})$$

$$\underline{r} = r_k \underline{U}$$

$$\underline{\dot{r}} = r_k \underline{U} + (r\dot{v})_k \underline{V}$$

Size Tests

* If $(4-5 \sin^2 i_o) < 10^{-3}$ disregard term

** If $(4-5 \sin^2 i_o)^2 < 10^{-3}$ disregard term

† If $e < 10^{-3}$ expand the quantity

$$\frac{(1-e^2)^{3/2} - 1}{e} = -\frac{3}{2}\, e \,(1 - \frac{1}{4} e^2)$$

†† If $e < 10^{-3}$ expand the quantity

$$\frac{1 - \sqrt{1-e_L^2}}{e_L^2} = \frac{1}{2} \quad (1 + \frac{1}{4} \quad e_L^2)$$

††† If $e < 10^{-3}$ expand the quantity

$$\frac{1 - (1-e_L^2)^{3/2}}{e_L^2} = \frac{3}{2}\,(1 - \frac{1}{4} \quad e_L^2)$$

2.2.6 Deviations from the Nominal

Determine the radial, transverse, and orthogonal components of the deviation in position from mean elements for the nominal case (including all terms):

radial Δr

transverse $r \Delta \theta_3 = r \Delta u + r \cos i_{oL} \Delta\Omega$

orthogonal $r \Delta \theta_1 = r \sin u \,\Delta i - r \sin i_{oL} \cos u \,\Delta\Omega$

$$|\Delta \underline{r}| = [(\Delta r)^2 + r^2 (\Delta \theta_3)^2 + r^2 (\Delta\theta_1)^2]^{1/2}$$

2.2.7 Subsatellite Point

As each point is calculated, the position obtained is used in the subsatellite point computation for output.

$$h = r - 1 + \left(\frac{3}{2} f^2 + f\right) U_z^2 - \frac{3}{2} f^2 U_z^4 \text{ converted to kilometers}$$

$$\text{Latitude} = \tan^{-1} \frac{U_z}{\sqrt{1-U_z^2 (1-f)^2}} \quad \text{converted to degrees}$$

$$\text{Longitude} = \tan^{-1} (y/x) - \theta_{gr_o} - \dot{\theta}(t-t_o) \text{ converted to degrees}$$

where $\dot{\theta} = 4.375269511 \times 10^{-3}$ radians per minute

and $\theta_{gr_o} = \theta_{gr_{oo}} + 0.9856472D + 360.9856472F$ converted to radians

where $\theta_{gr_{oo}}$ is the Greenwich sidereal time at the start of the epoch year in degrees, and D and F are the days and fractions of a day, respectively, from the start of the epoch year to the epoch.

2.2.8 Position-Error Analysis

The position-error analysis applies the above formulas 2.2.6 to the computed differences between the nominal orbit values and the experimental (omitted term) case. The subscript n refers to nominal case quantities.

$$\Delta r = r_{kn} - r_k$$

$$\Delta u = u_{kn} - u_k$$

$$\Delta\Omega = \Omega_{kn} - \Omega_k$$

$$\Delta i = i_{kn} - i_k$$

$$r\Delta\theta_3 = r_n \; \Delta u + r_n \cos i_{oL_n} \Delta\Omega$$

$$r\Delta\theta_1 = r_n \sin u_n \; \Delta i - r_n \sin i_{oL_n} \cos u_n \; \Delta\Omega$$

$$|\Delta \underline{r}\,| = \left[(\Delta r)^2 + r^2 (\Delta\theta_3)^2 + r^2 (\Delta\theta_1)^2 \right]^{1/2}$$

2.3 DIFFERENTIAL CORRECTION FORMULATION

The purpose of the differential correction is to relate the topocentric observation residuals to improvements in the orbit parameters. The formulation used in this program is similar (except for minor modifications) to that appearing in Aeronutronic Publication U-880*, and is presented here for reference.

2.3.1 Compute θ_{gr_o}, the Epoch Greenwich Sidereal Time:

$$\theta_{gr_o} \text{ (deg)} = (\theta - 360) D + \dot{\theta} F + \theta_{gr_{oo}}$$

* Astrodynamic Analysis for the National Space Surveillance Control Center, Sections 5.1 and 5.2 of Appendix 4A, June 1, 1960.

Where $\dot{\theta}$ is the rotation rate of the Earth in deg/solar day, $\theta_{gr_{oo}}$ is the Greenwich sidereal time at the start of the epoch year in degrees, and D and F are the days and fractions of a day, respectively, from the start of the epoch year to the epoch.

2.3.2 Station Vector

Compute the station vector $\underline{R}$ (X.Y,Z,) from the given quantities, ϕ, the geodetic latitude; λ_E, the east longitude; H, the altitude; and t, the time of observation:

$$C = \frac{1}{\sqrt{1 - (e^2 \sin^2 \phi)}}$$

$$e^2 = 2f - f^2$$

where $f = \frac{1}{298.3}$ is the Earth's flattening

$$S = C\ (1-e^2)$$

$$\theta_{(rad)} = \dot{\theta}\,(t-t_o) + \theta_{gr_o} + \lambda_E$$

where $\dot{\theta}$ is the rotation rate of the Earth in radians/solar minute.

$$X = -\ (C+H)\ \cos\phi\ \cos\ \theta$$

$$Y = -\ (C+H)\ \cos\phi\ \sin\ \theta$$

$$Z = -\ (S+H)\ \sin\ \phi$$

H has units of equatorial Earth radii

where X, Y, and Z are the components of the station vector $\underline{R}$.

2.3.3 Compute the Partial Differential Coefficients R_i and U_i:

(a) $R_u = \frac{a^2}{r}\ e \sin E$

(b) $R_n = -\frac{2}{3} r + (U-U_o) R_u$

(c) $R_{xN} = \frac{a^2}{r} [a_{xN} - \cos (E + \omega)]$

(d) $R_{yN} = \frac{a^2}{r} [a_{yN} - \sin (E + \omega)]$

(e) $R_c = \frac{2R_n}{n} (U-U_o)$

(f) $R_d = \frac{3R_n}{n^2} (U-U_o)^2$

(g) $U_u = \frac{a^2}{r} \sqrt{1-e^2}$

(h) $U_n = (U-U_o) U_u$

(i) $$U_{xN} = \frac{a^2}{r} \left\{ (1 + \frac{r}{a}) \sin (E + \omega) + a_{xN} e \sin E \right.$$

$$\left. \times \left[\frac{e^2 - (1 + \sqrt{1-e^2}) e \cos E}{\sqrt{1-e^2} (1 + \sqrt{1-e^2})^2} - \frac{a_{yN}}{1+\sqrt{1-e^2}} \right] \right\}$$

(j) $$U_{yN} = \frac{a^2}{r} \left\{ - (1 + \frac{r}{a}) \cos (E + \omega) + a_{yN} e \sin E \right.$$

$$\left. \times \left[\frac{e^2 - (1 + \sqrt{1-e^2}) e \cos E}{\sqrt{1-e^2} (1 + \sqrt{1-e^2})^2} \right] + \frac{a_{xN}}{1 + \sqrt{1-e^2}} \right\}$$

(k) $U_c = \frac{2U_n}{n} \quad (U - U_o)$

(l) $U_d = \frac{3U_n}{n^2} \quad (U - U_o)^2$

2.3.4 Topocentric Position of the Satellite

Compute the topocentric position of the satellite at the given time from the geocentric position $\underline{r}$ and the station position vector $\underline{R}$:

(a) $\underline{\rho_c} = \underline{r} + \underline{R}$

(b) $\rho_c = \sqrt{\underline{\rho_c} \cdot \underline{\rho_c}}$

(c) $\underline{L}_c = \frac{\underline{\rho_c}}{\rho_c}$

where the subscript c denotes computed quantities.

2.3.5 Slant Range Observations

If ρ, the slant range, is observed, compute the residual $\Delta\rho = \rho_o - \rho_c$, where the o subscript denotes observed quantities, and form the partial differential coefficients:

$$\frac{C_{\Delta n}}{n} = \underline{L}_c \cdot \underline{U}\, R_n + \underline{L}_c \cdot \underline{V}\, U_n$$

$$C_{\Delta a_{xN}} = \underline{L}_c \cdot \underline{U}\, R_{xN} + \underline{L}_c \cdot \underline{V}\, U_{xN}$$

$$C_{\Delta a_{yN}} = \underline{L}_c \cdot \underline{U}\, R_{yN} + \underline{L}_c \cdot \underline{V}\, U_{yN}$$

$$C_{\Delta U_o} = \underline{L}_c \cdot \underline{U}\, R_u + \underline{L}_c \cdot \underline{V}\, U_u$$

$$C_{\Delta\Omega} = \underline{L}_c \cdot \underline{V}\, r \cos i - \underline{L}_c \cdot \underline{W}\, r \sin i \cos u$$

$$C_{\Delta i} = \underline{L}_c \cdot \underline{W}\, r \sin u$$

$$C_{\Delta c''} = \underline{L}_c \cdot \underline{U}\, R_c + \underline{L}_c \cdot \underline{V}\, U_c$$

$$C_{\Delta d} = \underline{L}_c \cdot \underline{U}\, R_d + \underline{L}_c \cdot \underline{V}\, U_d$$

Enter the following linear correction equation into the system of such equations:

$$\Delta\rho = C_{\frac{\Delta n}{n}} \frac{\Delta n_o}{n_o} + C_{\Delta a_{xN}} \Delta a_{xN_o} + C_{\Delta a_{yN}} \Delta a_{yN_o} + C_{\Delta U_o} \Delta U_o$$

$$+ C_{\Delta\Omega} \Delta\Omega + C_{\Delta i} \Delta i + C_{\Delta c''} \Delta c'' + C_{\Delta d} \Delta d$$

2.3.6 Altazimuth Observations

If the azimuth, A, and elevation angle, h, are observed, compute the unit vectors $\underline{S}$, $\underline{E}$, and $\underline{Z}$:

$$\text{(a)} \quad \left.\begin{aligned} S_x &= \sin\phi\cos\theta \\ S_y &= \sin\phi\sin\theta \\ S_z &= -\cos\phi \end{aligned}\right\} \quad \underline{S}\text{, Southward Unit Vector}$$

$$\text{(b)} \quad \left.\begin{aligned} E_x &= -\sin\theta \\ E_y &= \cos\theta \\ E_z &= 0 \end{aligned}\right\} \quad \underline{E}\text{, Eastward Unit Vector}$$

$$\text{(c)} \quad \left.\begin{aligned} Z_x &= \cos\phi\cos\theta \\ Z_y &= \cos\phi\sin\theta \\ Z_z &= \sin\phi \end{aligned}\right\} \quad \underline{Z}\text{, Zenithal Unit Vector}$$

After this, compute $\underline{L}_{h_o}$, $\underline{\tilde{A}}_{h_o}$ and $\underline{\tilde{D}}_{h_o}$, where the o subscript denotes quantities which are calculated from observed data:

$$\text{(d)} \quad \left.\begin{aligned} L_{xh_o} &= -\cos A\cos h \\ L_{yh_o} &= \sin A\cos h \\ L_{zh_o} &= \sin h \end{aligned}\right\} \quad \underline{L}_{h_o}$$

(e) $$\left.\begin{aligned} \tilde{A}_{xh_o} &= \sin A \\ \tilde{A}_{yh_o} &= \cos A \\ \tilde{A}_{zh_o} &= 0 \end{aligned}\right\} \quad \underline{\tilde{A}}_{h_o}$$

(f) $$\left.\begin{aligned} \tilde{D}_{xh_o} &= \cos A \sin h \\ \tilde{D}_{yh_o} &= -\sin A \sin h \\ \tilde{D}_{zh_o} &= \cos h \end{aligned}\right\} \quad \underline{\tilde{D}}_{h_o}$$

Next rotate the components of $\underline{L}_{h_o}$, $\underline{\tilde{A}}_{h_o}$, $\underline{\tilde{D}}_{h_o}$ to the equatorial coordinate system:

(g) $$\underline{L}_o = L_{xh_o} \underline{S} + L_{yh_o} \underline{E} + L_{zh_o} \underline{Z}$$

(h) $$\underline{\tilde{A}}_o = \tilde{A}_{xh_o} \underline{S} + \tilde{A}_{yh_o} \underline{E} + \tilde{A}_{zh_o} \underline{Z}$$

(i) $$\underline{\tilde{D}}_o = \tilde{D}_{xh_o} \underline{S} + \tilde{D}_{yh_o} \underline{E} + \tilde{D}_{zh_o} \underline{Z}$$

Compute $\Delta \underline{L} = \underline{L}_o - \underline{L}_c$

Form the partial differential coefficients as in (2.3.5) with $\tilde{\underline{A}}_o$ replacing $\underline{L}_c$ and enter the following linear correction equation into the system of such equations:

$$\rho_c \, (\tilde{\underline{A}}_o \cdot \Delta \underline{L}) = C_{\frac{\Delta n}{n}} \frac{\Delta n_o}{n_o} + C_{\Delta a_{xN}} \Delta a_{xN_o} + C_{\Delta a_{yN}} \Delta a_{yN_o} + C_{\Delta U_o} \Delta U_o$$

$$+ C_{\Delta \Omega} \Delta\Omega + C_{\Delta i} \Delta i + C_{\Delta c''} \Delta c'' + C_{\Delta d} \Delta d$$

Again form the coefficients as in (2.3.5), this time with $\tilde{\underline{D}}_o$ replacing $\underline{L}_c$ and enter the following linear correction equation into the system of such equations:

$$\rho_c \, (\tilde{\underline{D}}_o \cdot \Delta \underline{L}) = C_{\frac{\Delta n}{n}} \frac{\Delta n_o}{n_o} + C_{\Delta a_{xN}} \Delta a_{xN_o} + C_{\Delta a_{yN}} \Delta a_{yN_o} + C_{\Delta U_o} \Delta U_o$$

$$+ C_{\Delta \Omega} \Delta\Omega + C_{\Delta i} \Delta i + C_{\Delta c''} \Delta c'' + C_{\Delta d} \Delta d$$

2.3.7 Equatorial Angular Coordinates

If α the topocentric right ascension, and δ the topocentric declination are observed, compute the vectors to $\underline{L}$, $\underline{A}$, and $\underline{D}$:

$$\text{(a)} \quad \left.\begin{aligned} L_{x_o} &= \cos\delta \cos\alpha \\ L_{y_o} &= \cos\delta \sin\alpha \\ L_{z_o} &= \sin\delta \end{aligned}\right\} \underline{L}_o$$

$$\text{(b)}\quad \left.\begin{aligned} A_{x_o} &= -\sin\alpha \\ A_{y_o} &= \cos\alpha \\ A_{z_o} &= 0 \end{aligned}\right\}\quad \underline{A}_o$$

$$\text{(c)}\quad \left.\begin{aligned} D_{x_o} &= -\sin\delta\cos\alpha \\ D_{y_o} &= -\sin\delta\sin\alpha \\ D_{z_o} &= \cos\delta \end{aligned}\right\}\quad \underline{D}_o$$

Compute $\Delta\underline{L} = \underline{L}_o - \underline{L}_c$, then form the partial differential coefficients and compute the linear correction euqation as in (2.3.6) substituting $\underline{A}_o$ for $\tilde{\underline{A}}_o$ and $\underline{D}_o$ for $\tilde{\underline{D}}_o$.

2.3.8 Range-Rate Observations

(a) If $\dot{\rho}$, the slant range rate, is observed, compute the station velocity $\dot{\underline{R}}$ and then $\dot{\rho}_c$:

$$\left.\begin{aligned} \dot{X} &= -Y\dot{\theta} \\ \dot{Y} &= X\dot{\theta} \\ \dot{Z} &= 0 \end{aligned}\right\}\quad \dot{\underline{R}}$$

where $\dot{\theta} = 0.058{,}834{,}47$

$$\dot{\underline{\rho}}_c = \dot{\underline{r}} + \dot{\underline{R}}$$

$$\dot{\rho}_c = \underline{L}_c \cdot \dot{\underline{\rho}}_c$$

(b) Compute $ex_\omega = a\,(e \cos E - e^2)$

and $ey_\omega = a\sqrt{1-e^2}\;\; e \sin E$

(c) Compute the partial differential coefficients $\dot{R}_i$ and $\dot{U}_i$:

(1) $\dot{R}_u = \sqrt{\mu}\; a^{3/2}\;\; ex_\omega r^{-3}$

(2) $\dot{R}_n = \frac{\dot{r}}{3} + (U - U_o)\; \dot{R}_u$

(3) $\dot{R}_{xN} = (\sqrt{\mu} a^{5/2}\;\; r^{-3})\;\; [\sin (E+\omega) - a_{xN}\; e \sin E - a_{yN}\,]$

(4) $\dot{R}_{yN} = (\sqrt{\mu} a^{5/2}\;\; r^{-3})\; [-\cos (E+\omega) - a_{yN}\; e \sin E + a_{xN}\,]$

(5) $\dot{R}_c = \frac{2\dot{R}_n}{n}\;\; (U - U_o)$

(6) $\dot{R}_d = \frac{3\dot{R}_n}{n^2}\;\; (U - U_o)^2$

(7) $\dot{U}_u = -\sqrt{\mu}\; a^{3/2}\; ey_\omega\;\; r^{-3}$

(8) $\dot{U}_n = \frac{r\dot{v}}{3} + (U - U_o)\; \dot{U}_u$

(9) $\dot{U}_{xN} = (\sqrt{\mu}\;\; a^{5/2}\; r^{-3})\;\; \sqrt{1-e^2}\; [\cos\; (E+\omega) - a_{xN}\; (1 + \frac{r^2}{ap}\,)\,]$

(10) $\dot{U}_{yN} = (\sqrt{\mu}\;\; a^{5/2}\; r^{-3})\;\; \sqrt{1-e^2}\;\; [\sin\; (E+\omega) - a_{yN}\; (1 + \frac{r^2}{ap}\,)\,]$

(11) $\dot{U}_c = \frac{2\dot{U}_n}{n}\; (U - U_o)$

(12) $\dot{U}_d = \frac{3\dot{U}_n}{n^2}\; (U - U_o)^2$

(d) Form the partial coefficients C_i:

$$C_{\frac{\Delta n}{n}} = \underline{L}_c \cdot \underline{U} [\rho_c (\dot{R}_n - \dot{v} U_n) - \dot{\rho}_c R_n] + \dot{\underline{\rho}}_c \cdot \underline{U} R_n$$

$$+ \underline{L}_c \cdot \underline{V} [\rho_c (\dot{U}_n + \frac{\dot{r}}{r} U_n) - \dot{\rho}_c U_n] + \dot{\underline{\rho}}_c \cdot \underline{V} U_n$$

$$C_{\Delta a_{xN}} = \underline{L}_c \cdot \underline{U} [\rho_c (\dot{R}_{xN} - \dot{v} U_{xN}) - \dot{\rho}_c R_{xN}] + \dot{\underline{\rho}}_c \cdot \underline{U} R_{xN}$$

$$+ \underline{L}_c \cdot \underline{V} [\rho_c (\dot{U}_{xN} + \frac{\dot{r}}{r} U_{xN}) - \dot{\rho}_c U_{xN}] + \dot{\underline{\rho}}_c \cdot \underline{V} U_{xN}$$

$$C_{\Delta a_{yN}} = \underline{L}_c \cdot \underline{U} [\rho_c (\dot{R}_{yN} - \dot{v} U_{yN}) - \dot{\rho}_c R_{yN}] + \dot{\underline{\rho}}_c \cdot \underline{U} R_{yN}$$

$$+ \underline{L}_c \cdot \underline{V} [\rho_c (\dot{U}_{yN} + \frac{\dot{r}}{r} U_{yN}) - \dot{\rho}_c U_{yN}] + \dot{\underline{\rho}}_c \cdot \underline{V} U_{yN}$$

$$C_{\Delta U_o} = \underline{L}_c \cdot \underline{U} [\rho_c (\dot{R}_u - \dot{v} U_u) - \dot{\rho}_c R_u] + \dot{\underline{\rho}}_c \cdot \underline{U} R_u$$

$$+ \underline{L}_c \cdot \underline{V} [\rho_c (\dot{U}_u + \frac{\dot{r}}{r} U_u) - \dot{\rho}_c U_u] + \dot{\underline{\rho}}_c \cdot \underline{V} U_u$$

$$C_{\Delta\Omega} = - \underline{L}_c \cdot \underline{U} \rho_c r \dot{v} \cos i + \underline{L}_c \cdot \underline{V} \cos i [\rho_c \dot{r} - \dot{\rho}_c r]$$

$$+ \dot{\underline{\rho}}_c \cdot \underline{V} r \cos i + \underline{L}_c \cdot \underline{W} \sin i [\rho_c (r \dot{v} \sin u - \dot{r} \cos u)$$

$$+ \dot{\rho}_c r \cos u] - \dot{\underline{\rho}}_c \cdot \underline{W} r \sin i \cos u$$

$$C_{\Delta i} = \underline{L}_c \cdot \underline{W} \left[\rho_c \left(r \dot{v} \cos u + \dot{r} \sin u \right) - \dot{\rho}_c \, r \sin u \right]$$

$$+ \dot{\underline{\rho}}_c \cdot \underline{W} \, r \sin u$$

$$C_{\Delta c''} = \underline{L}_c \cdot \underline{U} \left[\rho_c \left(\dot{R}_c - \dot{v} \, U_c \right) - \dot{\rho}_c R_c \right] + \dot{\underline{\rho}}_c \cdot \underline{U} \, R_c$$

$$+ \underline{L}_c \cdot \underline{V} \left[\rho_c \left(\dot{U}_c + \frac{\dot{r}}{r} U_c \right) - \dot{\rho}_c U_c \right] + \dot{\underline{\rho}}_c \cdot \underline{V} \, U_c$$

$$C_{\Delta d} = \underline{L}_c \cdot \underline{U} \left[\rho_c \left(\dot{R}_d - \dot{v} \, U_d \right) - \dot{\rho}_c R_d \right] + \dot{\underline{\rho}}_c \cdot \underline{U} \, R_d$$

$$+ \underline{L}_c \cdot \underline{V} \left[\rho_c \left(\dot{U}_d + \frac{\dot{r}}{r} U_d \right) - \dot{\rho}_c U_d \right] + \dot{\underline{\rho}}_c \cdot \underline{V} \, U_d$$

(e) Compute $\Delta \dot{\rho} = \dot{\rho}_o - \dot{\rho}_c$, the slant range-rate residuals

(f) Enter the following linear correction equation into the system of such equations:

$$\rho_c \, \Delta \dot{\rho} = C_{\frac{\Delta n}{n}} \frac{\Delta n_o}{n_o} + C_{\Delta a_{xN}} \Delta a_{xN_o} + C_{\Delta a_{yN}} \Delta a_{yN_o} + C_{\Delta U_o} \Delta U_o$$

$$+ C_{\Delta \Omega} \, \Delta \Omega_o + C_{\Delta i} \, \Delta i + C_{\Delta c''} \, \Delta c'' + C_{\Delta d} \, \Delta d$$

2.3.9 Rejection of Observations

In the normal mode of operation, this program rejects observations in the same manner as the operational differential correction program (SGPDC).* The absolute maxima for residuals of range and angle observations are set to 1000 km and for range-rate observations to 0.5 km/sec during the initial iterations in the differential correction process. The relative maxima are computed as the product of a constant (1.5) times the computed rms errors for the combined range, angle and range-rate observations. (The rms is computed as the square root of the sum of the squares of the accepted residuals divided by the square root of the number of accepted residuals.) When the rms of the positional observations become less than 50 km., the absolute rejection criterion is reduced to 75 km.

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* Aeronutronic Publication U-1691, revised 1 October 1962.

In the override mode, where a specified set of elements is corrected "n" times regardless of convergence (see fields 1 and 2 on D.C. control card Figure 27), the observation rejection process is specified on the D.C. control card (field 9) in one of two ways:

(1) Absolute maximum rms multiplier method: An initial absolute maximum for range and angle observations and an rms multiplier for each correction are specified.

(2) Absolute maximum method: An absolute maximum for range and angle observations is entered for each correction specified.

In either case, the rejection of range-rate observations is treated as in the operational D.C. program.

A further modification to the rejection process is available in both the normal and override modes. This modification is, the time factor, $\frac{t - t_o}{3}$, (where $t - t_o$ is in units of days) multiplying all absolute maxima prior to the rejection test. This factor is not used if it is less than unity. This option is specified in field seven of the D.C. control card (Figure 27).

2.3.10 Corrected Elements

Compute the corrected elements L_o, a_{xN_o}, a_{yN_o}, h_{x_o}, h_{y_o}, h_{z_o}, c",d

(a) Let $\sum_{i=1}^{N} C_{ij} \Delta_i = v_j$, j = 1,2,3,...represent all of the linear correction equations (i.e., the C_i's are the coefficients, the Δ_i's are the corrections to the orbital parameters at time t_o, the v's are the observation residuals, and N is the number of parameters being corrected)

Statistical weighting of the input observations is accomplished in this program by assigning a constant standard deviation, σ, for each quantity that is observed by a particular sensor. These standard deviations are entered into the linear correction equations is shown below. They are introduced to the program by the SIGMA cards (see figure 14). The standard deviation for each observation may be varied by entering four multiplying factors on each observation card. (see figure 28).

The following matrix equation is solved to give the corrections, in the least squares sense, to the orbital parameters at time t_o. σ_j is the assumed standard deviation of the observation producing v_j.

$$\begin{bmatrix} \sum_j C_{1j}^2 \sigma_j^{-2} & \sum_j C_{1j}C_{2j}\sigma_j^{-2} & \cdots & \sum_j C_{1j}C_{Nj}\,\sigma_j^{-2} \\ \sum_j C_{1j}C_{2j}\,\sigma_j^{-2} & \sum_j C_{2j}^2\,\sigma_j^{-2} & \cdots & \sum_j C_{2j}C_{Nj}\,\sigma_j^{-2} \\ \vdots & \vdots & & \vdots \\ \sum_j C_{1j}C_{Nj}\,\sigma_j^{-2} & \sum_j C_{2j}C_{Nj}\sigma_j^{-2} & \cdots & \sum_j C_{Nj}^2\sigma_j^{-2} \end{bmatrix} \begin{bmatrix} \Delta_1 \\ \Delta_2 \\ \vdots \\ \Delta_N \end{bmatrix} = \begin{bmatrix} \sum_j C_{1j}v_j\sigma_j^{-2} \\ \sum_j C_{2j}v_j\sigma_j^{-2} \\ \vdots \\ \sum_j C_{Nj}v_j\sigma_j^{-2} \end{bmatrix}$$

(b) The resulting corrections are applied as follows (a prime means that the element is a corrected element);

$$n_o' = n_o \left(1 + \frac{\Delta n_o}{n_o}\right)$$

$$(c'')' = c'' + \Delta c''$$

$$d' = d + \Delta d$$

$$c_o' = -\frac{(c'')'}{n_o'^2} \cdot \frac{\pi^2}{360}$$

$$U_o' = U_o + \Delta U_o$$

$$a'_{xN_o} = a_{xN_o} + \Delta a_{xN_o}$$

$$a'_{yN_o} = a_{yN_o} + \Delta a_{yN_o}$$

$$\Omega'_o = \Omega_o + \Delta\Omega$$

$$i' = i + \Delta i$$

$$L'_o = U'_o + \Omega'_o \qquad \text{If } W'_z = \cos i' \geqslant 0$$

$$L'_o = U'_o - \Omega'_o \qquad \text{if } W'_z = \cos i' < 0$$

$$e'^2_o = a'^2_{xN_o} + a'^2_{yN_o}$$

$$a'_o = \left(\frac{k_e\sqrt{\mu}}{n'_o}\right)^{2/3} \left[1 - \frac{1}{2} J_2 \frac{a_e^2}{p^2} \left(1 - \frac{3}{2}\sin^2 i'\right)\sqrt{1 - e^2}\right]$$

$$p'_o = a'_o \left(1 - e'^2_o\right)$$

$$\left.\begin{aligned} W'_{x_o} &= \sin\Omega'_o \sin i' \\ W'_{y_o} &= -\cos\Omega'_o \sin i' \\ W'_{z_o} &= \cos i' \end{aligned}\right\} \underline{W}'_o$$

$$\left.\begin{aligned} h'_{x_o} &= \sqrt{p'_o}\, W'_{x_o} \\ h'_{y_o} &= \sqrt{p'_o}\, W'_{y_o} \\ h'_{z_o} &= \sqrt{p'_o}\, W'_{z_o} \end{aligned}\right\} \qquad \underline{h}'_o$$

$$\Omega'_o = \tan^{-1} \frac{W'_{x_o}}{-W'_{y_o}}$$

$$\omega'_o = \tan^{-1} \frac{a'_{yN_o}}{a'_{xN_o}}$$

where the quadrant is determined from the signs of the numerator and denominator

SECTION 3

PROGRAM OPERATION

3.1 GENERAL DESCRIPTION

The equations appearing in Section 2 are combined in a Philco 2000 computer program. The program is divided into three main sections: a differential correction subroutine, an ephemeris calculation subroutine, and an ephemeris calculation subroutine provided with an error analysis section. With the program options available, these sections can be used in any combination or order.

When operating this program, the following tapes must be used as shown in the table, regardless of the section of the program being used.

Table III Tape Locations

Tape	Location (Logical tape unit)
Output tape*	5
Standard library tape	7
Input tape	8
SEAI tape (if needed)	10

*NOTE: Use data select 9 to print output from ephemeris section of program. Use data select 0 to print output from differential correction section of program.

3.1.1 Program Input

As input, the program expects the $\underline{N}$, $\underline{M}$ elements, L_o, a_{xN_o}, a_{yN_o}, and $\underline{h}_o$; the drag parameters c_o and d; the area to mass ratio $\frac{a\gamma}{m}$; plus other data, depending on the program option chosen. This is discussed more fully in Section 3.3.

3.1.2 Program Output (See Section 3.3 for samples of output)

The output obtained depends upon which section of the program is being used and also upon several output options.

(a) If differential correction is being performed, the program will print out the correction residuals, the old elements and the new (corrected) elements, along with many auxiliary quantities. The program will also punch out a set of corrected element cards. If the output option IØUT=1 is specified with differential correction, the program will output many quantities calculated in the ephemeris subroutine. These include the components of $\underline{r}$ and $\dot{\underline{r}}$; the secular, long-period, short-period, and overall changes in the elements; the values of the 89 general perturbation terms (see section 3.6 for units of terms) and also many intermediate quantities.

(b) If ephemeris calculation is being performed, the program will output $r\Delta\theta_1$, $r\Delta\theta_3$, $|\Delta\underline{r}|$, Δr, and the quantities specified by the output option IØPT (see summary of program options, Section 3.2) after each time increment from t_o to t_{END}.

(c) If ephemeris calculation with position-error-analysis is being performed, the program compares the results obtained by using all of the terms with the results obtained when terms selected by the operator are omitted. The program then outputs the following quantities: Δr, $r\Delta\theta_3$, $r\Delta\theta_1$, $|\Delta\underline{r}|$, $\Delta\Omega$, Δu and Δi (all calculated as in Section 2.2.8) and the quantities specified by the output option IØPT after each time increment from t_o to t_{END}.

3.2 PROGRAM OPTIONS

The following outline is a summary of the options available to the user of this computer program. The application of these options is discussed more fully in Section 3.3.

3.2.1 ISENT

(a) ISENT = +1: Input sensor data with cards and then input SIGMA cards (Note: This must be +1 whenever the D. C. Program gets its sensor data from cards, even if an ephemeris calculation appears first.)

(b) ISENT = -1: Bypass SIGMA card input and:

(1) For D. C. option, input sensor data from tape

(2) For ephemeris option, this indicates no sensor data input

(c) ISENT = 0: (Blank card) Input SIGMA cards and input sensor data from tape

3.2.2 IDCEPH

(a) IDCEPH = 0: End program (Use blank card for 0)

(b) IDCEPH = +1: Go to differential correction subroutine

(c) IDCEPH = -1: Go to ephemeris calculation

3.2.3 ICAL: Used Only Under Ephemeris Option: IDCEPH = -1

(a) ICAL = 1: Go to ephemeris calculation without error analysis

(b) ICAL = 2: Go to ephemeris calculation with error analysis

3.2.4 IØPT: Used only under ephemeris option: IDCEPH = -1

(a) IØPT = 1: Output $t, x, y, z, \dot{x}, \dot{y}, \dot{z}$

(b) IØPT = 2: Output t, Lat., Long., H

(c) IØPT = 3: Output $t, x, y, z, \dot{x}, \dot{y}, \dot{z}$ and t, Lat., Long., H

(d) IØPT = 4: Output t, x, y, z, $\dot{x}$, $\dot{y}$, $\dot{z}$ and term values from General Perturbations section of XYZ Subroutine

(e) IØPT = 5: Output t, Lat., Long., H and term values from General Perturbations section of XYZ Subroutine

(f) IØPT = 6: Output t, x, y, z, $\dot{x}$, $\dot{y}$, $\dot{z}$ and t, Lat., Long., H and term values from General Perturbations section of XYZ Subroutine

3.2.5 <u>IBACK1</u>: Used Only Under Ephemeris Option: IDCEPH = -1

(a) IBACK1 = 1: Input new time values and compute another ephemeris using the same General Perturbations terms specified by NTERMS(I) (Note: If IBACK1 = 1 is used after the Error Analysis Section, the program will input a new omitted term case and compare this with the same nominal term case used with the preceding error analysis.)

(b) IBACK1 = 2: Input new times and terms and compute another ephemeris

(c) IBACK1 = 3: Input a new value for IDCEPH to determine whether to end program or start another case with new elements

(d) IBACK1 = 4 : End program

3.2.6 <u>IØUT</u>

(a) IØUT = 0: There will be <u>no</u> XYZ output from D.C. Subroutine, only D.C. output

(b) IØUT = 1: There will be XYZ output from D.C. Subroutine after each batch of observations

3.2.7 <u>AGØM</u>

(a) AGØM $\geq$ 1: Include calculation of radiation pressure effects

(b) AGØM < 1: Omit calculation of radiation pressure effects

3.2.8 DTERM

(a) DTERM = 0: Calculate DTERM in the BEGIN subroutine

(b) DTERM $\neq$ 0: Use this value for d and bypass the calculation of DTERM

3.2.9 TERMS (Two cards: Columns 1 through 80 on the first card correspond to terms Q(1) through Q(80) in the program. Columns 1 through 9 on the second card correspond to terms Q(81) through Q(89) in the program.)

(a) A one punched in any of the above mentioned columns will cause the corresponding term to be included in the ephemeris calculation.

(b) A zero (or no punch) punched in any of the above mentioned columns will cause the corresponding term to be set equal to zero in the ephemeris calculation.

(c) Identification of terms Q(1) through Q(89) is found in Section 3.6.

3.2.10 For description of further options see Differential Correction Control Card format (Figure 27)

3.3 EXAMPLES OF PROGRAM OPERATION

The data, the type of cards contained in the input deck and also the output obtained from the program depend upon the program options chosen. Therefore, this section is divided into three subsections. Each subsection describes one sample case.

The understanding of the following descriptions will be facilitated by referring to the flow chart of the main program (Section 3.5).

3.3.1 SAMPLE CASE #1: Perform differential correction of elements with sensor data input from cards.

(a) Input cards used: The following is a description of the cards used and the reasons for their use, listed in the order in which the cards appear in the input deck. (see Figures 4 and 5)

(1) ISENT CARD (Figure 11) with a 1 punched in Column 6. This card tells the program to take the sensor data from cards (See Section 3.2.1).

(2) 3 SENSØR CARDS: These cards contain sensor data in standard SPADATS format, Figure 12. The three cards indicate that only three sensors are used for this case. (Note: no more than 200 sensor cards may be used)

(3) ENDSENS CARD (Figure 13): This card is used to tell the program that all SENSØR CARDS have been read in.

(4) ENDSIGMA CARD (Figure 15): This card is normally used after SIGMA CARDS, but in this case indicates that no SIGMA CARDS are used. (Note: no more than 100 sigma cards may be used)

(5) IDCEPH CARD (Figure 16): With a 1 punched in column 6. This card directs the program to the differential correction subroutine (see Section 3.2.2).

(6) 7 ELEMENT CARDS (see Figures 17 to 23): These cards contain the initial elements needed for the program. They are in standard SPADATS format. An E must appear in column 80 of each card. These standard elements are converted to the $\underline{N}$, $\underline{M}$ elements by the program.

(7) AGØM CARD (see Figure 24 and Section 3.2.7): The 0.0 in columns 8, 9 and 10 is the DTERM input. Since it is zero, it tells the program to calculate DTERM, the period decay acceleration.

The 1 in column 20 is the IØUT option. This tells the program to output data (see Section 3.2.6) from the XYZ subroutine.

The 0.5 in columns 29 and 30 is the value of $\frac{A\gamma}{m}$ and since it is less than 1, it causes the program to omit the calculation of radiation pressure perturbations.

(8) 2 TERMS CARDS (see Figures 25 and 26): These cards tell the program which of the 89 General Perturbations terms to include in the calculations. In this case, all 89 terms are included.

(9) D. C. CØNTRØL CARD (see Figure 27): The 1 in column 15 tells the program to output all residuals. The 0 in column 79 tells the program to omit use of σ's.

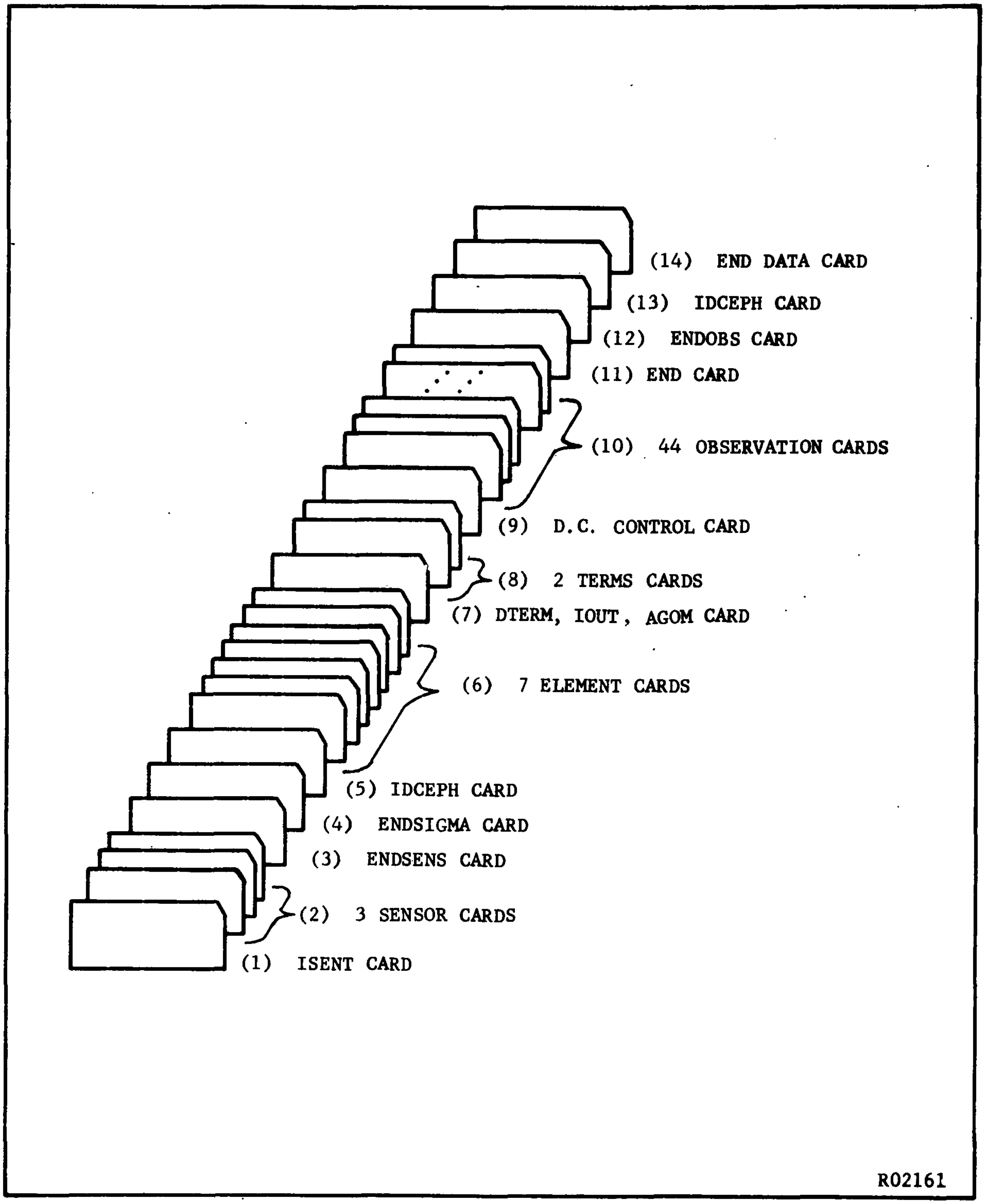

FIGURE 4. INPUT CARD DECK FOR SAMPLE CASE #1

CARDS	COLUMNS 1	13	25	37	49	61	72	80
(1) ISENT	1							
(2) SENSOR 1st CARD	0033+324241-	2534477 165	10 4 003301	ORGAN PASS N	M 131	9001 01		S
SENSOR 2nd CARD	0034-259597-	0282475 154	40 4 003401	OLIFANTSFONT	AIN SAFR601	9002 02		S
SENSOR 3rd CARD	0036+364638-	3537949 2	40 4 003601	SANFERNANDO	SPAIN 407	9004 04		S
(3) ENDSENS	ENDSENS							
(4) ENDSIGMA	ENDSIGMA							
(5) IDCEPH	1							
(6) ELEMENT CARD #1	020001 1 59	ETA 3		0.19025	33.335			E
ELEMENT CARD #2	020001 21959	26	8.114986		216.19899			E
ELEMENT CARD #3	020001 30.09	03815 21	0.560	167.08	1.08004228	27		E
ELEMENT CARD #4	020001 4-0.1	739400E-7						E
ELEMENT CARD #5	020001 5							E
ELEMENT CARD #6	020001 6							E
ELEMENT CARD #7	020001 7							E
(7) DTERM IOUT AGOM	0.0	1	0.5					
(8) TERMS CARD #1	111111111111	111111111111	111111111111	111111111111	111111111111	111111111111		11111111
TERMS CARD #2	111111111							
(9) D.C. CONTROL CARD		1						0
(10) OBSERVATION CARD#1	020170036090	918202431935	782078039408			3		
OBSERVATION CARD#2	020170036090	918202441540	80987K051261			3		
OBSERVATION CARD#3	020170036090	918202446342	82299K05719	9		3		
⋮								
OBSERVATION CARD#43	020170036090	925192505414	151732K23932	9		3		
OBSERVATION CARD#44	020170036090	925192521415	14361 2K25030	7		3		
(11) END	END							
(12) ENDOBS	ENDOBS							
(13) IDCEPH	0							
(14) END DATA	END DATA							

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FIGURE 5. INPUT CARD DATA FOR SAMPLE CASE NO. 1

(10) 44 ØBSERVATIØN CARDS (see Figure 28): These cards contain observation data in standard SPADATS format. The 44 cards indicate that the data from 44 observations are to be included in the differential correction.

(11) END CARD (see Figure 29): This must appear after each set of observations. In this case, there is only one set.

(12) ENDØBS CARD (see Figure 30): This card tells the program that there are no more sets of ØBSERVATIØN CARDS to read in.

(13) IDCEPH CARD with 0 punched in column 6. This card causes the program to end after the ENDØBS card is read in, which is after the differential correction of the elements.

(14) END DATA (see Figure 34): This card is used to tell the computer that all the data for this job has been read in.

(b.) Output from Sample case #1: Figure 6 shows the output obtained from sample case #1. This includes all of the quantities described in 3.1.2a above.

3.3.2 SAMPLE CASE #2: Perform ephemeris calculation without error Analysis. Output all quantities possible.

(a) Input cards used: The following is a description of the cards used and the reasons for their use, listed in the order in which the cards appear in the input deck. (see Figure 7)

(1) ISENT CARD (see Figure 11) with a -1 punched in columns 5 and 6. This card tells the program to bypass the input of the SENSØR and SIGMA cards since these are not needed for ephemeris calculation.

(2) IDCEPH CARD (see Figure 16): with a -1 punched in columns 5 and 6. This card directs the program to the ephemeris calculation section.

(3) 7 ELEMENT CARDS (see Figures 17 to 23): These cards contain the initial elements needed for the program. They are standard SPADATS format. An E must appear in column 80 for each card. These standard elements are converted to the N, M elements in the program.

(4) IØPT, ICAL CARD (see Figure 31): The 6 in column 6 is the IØPT indicator. Since it is 6 it will cause the program to output all quantities possible (see Section 3.2.4). The 1 in column 12 is the ICAL indicator. This directs the program to the ephemeris calculation without error analysis (see Section 3.2.3).

(5) 2 TERMS CARDS (see Figures 25 and 26 and Section 3.2.9): These cards tell the program which of the 89 General Perturbations terms to include in the ephemeris calculation. In this case, all 89 terms are included.

(6) Input for XYZ Subroutine (see Figure 32): The first six quantities are the epoch time and are punched as shown in Figure 32.

The 0.5 in columns 38, 39, and 40 in AGØM. This is the quantity $\frac{A\gamma}{m}$ and since it is less than 1, it causes the program to omit the calculation of the radiation pressure perturbations.

The 0.0 in columns 44, 45 and 46 is DTERM. Since it is zero, it tells the program to calculate DTERM, the period decay acceleration.

The 1000.0 in columns 55 through 60 is DELTAT. This is the time increment to be used in the ephemeris calculation. The 1000.0 in columns 67 through 72 is TEND. This is the time interval to be covered in the ephemeris calculation. Since TEND is equal to DELTAT, the program will calculate the ephemeris for only two times, t_o and t_o + 1000 minutes.

(7) IBACK1 CARD (see Figure 33 and Section 3.2.5) with a 4 punched in column 6. This card tells the program to end after the ephemeris calculation.

(8) END DATA CARD (see Figure 34): An END DATA CARD (as in Sample Case #1) must appear after the last card in the data deck.

(b) Output from Sample case #2

The program prints out the quantities described in Section 3.1.2b as shown by Figure 8. Only the output for $t = t_o$ + 1000 minutes is shown.

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SATELLITE NO. 000   SATELLITE NAME: 59 ETA 3   ELEMENT SET NO. 1   TIME OF EPOCH 268.1498599

CASE NO.	RMS KM.	RMS2 KM/SEC	DELTA N/N	DELTA AXN	DELTA AYN	DELTA UO	DELTA NODE	DELTA I	DELTA C*
0	.336538+1	.00000000	.20400669-6						

DELTA D	DELTA A	DELTA E	DELTA OMEGA
	-.18149-4	.0000000	.0000000

CORRECTED ELEMENTS

REV. NO.	CASE NO.	L DEGREES	TO DAYS	A EARTH RADII	E	I DEG.	NODE DEG.	OMEGA DEG.	CO DA/REV**2	PER ALT ST. MI.	PA MINUTES
	1	215.96485	268.14960	1.3337970	.19033	33.335	210.257	166.944	-.17593-7	316.7	130.149

AXN	AYN	HBAR	R KM	RDOT KM/SEC
-.18541	.04299	1.13380	10067.98624	-295.15066

Note: This is the output which appears on data select zero along with the output described on pp. 3 - 86 of the SPADATS manual.* The quantity "d" was not corrected in this sample case.

*Aeronutronic publication U-1691, revised 1 October, 1962.

FIGURE 6

```
          EXPERIMENTAL GENERAL PERTURBATIONS DIFFERENTIAL CORRECTION PROGRAM
   (THIS IS OUTPUT FROM THE  XYZ  SUBROUTINE WHEN USED BY THE DIFFERENTIAL CORRECTION SUBROUTINE)

SATELLITE NO. 000  SATELLITE NAME: 59 ETA 3   ELEMENT SET NO.   0              TIME OF EPOCH     268.1498599

      TIME
Y MM DD HH MM SS.SS
9 09 18 20 24 31.94

TERMS  (1)...(89)
   ( 1)           ( 2)           ( 3)           ( 4)           ( 5)           ( 6)           ( 7)           ( 8)
-3.9601247-005  2.9975328-009 -4.8216961-008 -3.2241484-010 -2.6105369-009  2.6992712-005  2.2319610-008  4.6594304-008

   ( 9)           (10)           (11)           (12)           (13)           (14)           (15)           (16)
 7.1796542-010  1.8100779-009  2.4417456-006  2.8465277-004 -1.8830216-004 -6.9110890-006 -9.2201417-006 -6.4955499-006

   (17)           (18)           (19)           (20)           (21)           (22)           (23)           (24)
-3.4868084-008  1.3193686-006 -2.3935809-007  3.5441744-007 -1.1641918-005  2.1041109-005  1.7860358-007 -1.2918852-006

   (25)           (26)           (27)           (28)           (29)           (30)           (31)           (32)
 8.2658256-007 -1.1480685-006 -6.1435217-006 -3.2978323-008  1.2478650-006 -2.8310050-007  4.1919207-007 -2.9485679-005

   (33)           (34)           (35)           (36)           (37)           (38)           (39)           (40)
-6.4901371-007  1.9900780-005  1.6135767-007 -1.2218711-006  9.7765181-007 -1.3578936-006 -2.9154630-009  4.4046220-010

   (41)           (42)           (43)           (44)           (45)           (46)           (47)           (48)
-3.6358744-010  1.9391165-005  2.5317143-008  3.9555058-010 -2.1622597-008 -8.0075907-010  7.1631472-006 -2.9232456-006

   (49)           (50)           (51)           (52)           (53)           (54)           (55)           (56)
-1.0131838-004 -3.1023594-005 -2.3039201-005  9.4693028-006 -1.4364794-007 -9.7948475-005  4.6532041-007 -1.8865990-004

   (57)           (58)           (59)           (60)           (61)           (62)           (63)           (64)
-6.0480068-004 -2.8411116-007 -1.2651875-006  2.2001719-005 -2.1343952-005 -2.1215354-005 -5.1215201-005 -9.4538048-006

   (65)           (66)           (67)           (68)           (69)           (70)           (71)           (72)
-4.9772979-007 -2.2317588-006  3.4597286-008  7.8937039-007  7.8761143-006 -5.1775219-005 -1.4065208-006 -1.6304391-005

   (73)           (74)           (75)           (76)           (77)           (78)           (79)           (80)
-1.9469181-007  6.5698518-005  2.5148801-004 -1.0541616-004 -2.5455340-006 -3.2911407-005  3.9838930-006  1.2331269-007

   (81)           (82)           (83)           (84)           (85)           (86)           (87)           (88)
 2.4341315-004  4.0595298-004 -3.7028329-005  2.5550854-005 -1.8162522-004 -2.7802816-005  2.7458959-006  1.9382550-004

   (89)
 7.6608795-007

    AXNS           AXNL           AXNSL          AYNS           AYNL           AYNSL          XNODES         XNODEL
                                                                                              RAD.           RAD.
-1.3803325-001  2.8604358-006 -1.3803839-001  1.3055250-001  4.7735828-004  1.3102985-001  4.0344121+000 -1.9920530-004


    ISUBL          ISUBOL           I             DELTI          ESUBL            X              Y              Z
    RAD.           RAD.            RAD.           RAD.                           E.R.           E.R.           E.R.
-9.7626803-005  5.8180714-001  5.8180714-001 -2.1668244-004  1.9031871-001  5.0170842-001 -8.8108988-001  6.2041845-001
    XDOT           YDOT           ZDOT           USUBK          DELTU          RSUBK          OMGASO         DELTRD
E.R./KEMIN     E.R./KEMIN     E.R./KEMIN         RAD.           RAD.           E.R.           RAD.       E.R./KEMIN
6.9822207-001  6.6209179-001  8.4816266-002  1.2525262+000 -7.9435211-004  1.1886715+000  2.3846397+000 -6.3710228-005
   DELTRV          RDOT           RVDOT          RDELT1         RDELT3         ABSDRB          DELTR          XNODESL
E.R./KEMIN     E.R./KEMIN     E.R./KEMIN          KM.            KM.            KM.            KM.            RAD.
1.8118672-004 -1.5179732-001  9.5395809-001 -2.3907659+000 -1.9826391+000  3.1059008+000 -5.2227253-001  4.0348508+000
   DELNOD           XL            XLSUBL         NODELPI        NODELM
    RAD.           RAD.            RAD.        RAD./MIN.      RAD./MIN.
6.3788866-004 -4.3421287+002 -1.4167177-004  1.9394455-005 -2.8385682-009
```

This is the output which appears on data select nine for each observation used in the differential correction section of the program. See section 3.6 for units and meaning of quantities shown above.

FIGURE 6 (cont)

FLOW CHART OF MAIN PROGRAM (CONTINUED)

CARDS	COLUMNS 1	13	25	37	49	61 – 72	80
(1) ISENT	-1						
(2) IDCEPH	-1						
(3) ELEMENT CARD #1	6660011 1 SAM	PLE		0.001	30.0		E
ELEMENT CARD #2	6660011 21959	26	8.149861		60.0		E
ELEMENT CARD #3	6660011 3		0.0	30	.01.02249421	7	E
ELEMENT CARD #4	6660011 4-0.1	5E-4					E
ELEMENT CARD #5	6660011 5						E
ELEMENT CARD #6	6660011 6						E
ELEMENT CARD #7	6660011 7						E
(4) IOPT, ICAL	6 1						
(5) TERMS CARD #1	111111111111	111111111111	111111111111	111111111111	111111111111	111111111111	11111111
TERMS CARD #2	1111111111						
(6) (SEE DESCRIPT.)	3 04	04 11	43 28.00	0.5 0.0	1000.0	1000.0	
(7) IBACK1	4						
(8) END DATA	END DATA						

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FIGURE 7. INPUT CARD DATA FOR SAMPLE CASE NO. 2

```
                    EXPERIMENTAL GENERAL PERTURBATIONS DIFFERENTIAL CORRECTION PROGRAM
                              (EPHEMERIS OUTPUT UNDER OUTPUT OPTION NO. 6)

SATELLITE NO. 666  SATELLITE NAME) SAMPLE     ELEMENT SET NO.  0               TIME OF EPOCH     268.1498599

      TIME
Y MM DD HH MM SS.SS
3  4  4 11 43 28.00

     T               X              Y              Z            XDOT          YDOT          ZDOT
    MIN.            E.R.           E.R.           E.R.       E.R./KEMIN    E.R./KEMIN    E.R./KEMIN
 1.0000000+003 -4.5917556-001 -7.7432636-001 -4.7016098-001  8.8383948-001 -4.1073781-001 -1.8499131-001

     T             XLAT           ELONG            H
    MIN.           DEG.           DEG.            KM.
 1.0000000+003 -2.7734669+001 -6.8515874+001  1.0419859+002

TERMS  (1)...(89)
    ( 1)           ( 2)           ( 3)           ( 4)           ( 5)           ( 6)           ( 7)           ( 8)
-9.6551684-005  2.4889470-008 -2.2105271-007 -3.2667593-014 -3.3157907-013  1.5297884-004  7.1625600-008  2.0207267-007

    ( 9)           (10)           (11)           (12)           (13)           (14)           (15)           (16)
 7.9371777-014  2.9513245-013  0.             0.             0.             0.             0.             0.

    (17)           (18)           (19)           (20)           (21)           (22)           (23)           (24)
 0.             0.             0.             0.             0.             0.             0.             0.

    (25)           (26)           (27)           (28)           (29)           (30)           (31)           (32)
 0.             0.             0.             0.             0.             0.             0.             0.

    (33)           (34)           (35)           (36)           (37)           (38)           (39)           (40)
 0.             0.             0.             0.             0.             0.            -2.1330407-008  5.1080821-014

    (41)           (42)           (43)           (44)           (45)           (46)           (47)           (48)
 7.9765461-015  5.6627160-005  9.6515070-008  4.6704224-014 -1.8980048-008 -3.6446622-014  0.             0.

    (49)           (50)           (51)           (52)           (53)           (54)           (55)           (56)
 0.             0.             0.             0.             0.             0.             0.            -3.8931520-004

    (57)           (58)           (59)           (60)           (61)           (62)           (63)           (64)
 1.3923981-006  5.6681263-015 -1.8522308-011  9.8424295-008  1.0262346-007  1.7562386-007 -4.7747178-005  8.7792641-008

    (65)           (66)           (67)           (68)           (69)           (70)           (71)           (72)
 1.1295279-014 -3.6767554-011 -1.6002937-015  0.             2.7910905-008 -9.0918462-005 -2.2060531-011  6.7903952-008

    (73)           (74)           (75)           (76)           (77)           (78)           (79)           (80)
-1.0536716-011  4.6377381-005  4.8844800-004 -1.4005154-004 -3.0206419-011  1.5458743-007  3.7157527-008 -3.1621316-011

    (81)           (82)           (83)           (84)           (85)           (86)           (87)           (88)
 4.7598615-004 -8.7695810-007 -1.4612218-007 -1.1849937-007 -2.4440431-004  1.7975421-007  2.5937480-008  5.6878916-004


    (89)
-2.8012287-013

   RDELT1         RDELT3         ABSDRW         DELTR
    KM.            KM.            KM.            KM.
 2.0446340+000  1.5227111-001  2.0502962+000 -3.0285926-001
```

This is the output obtained from data select nine from sample case number 2. The zero value of the terms shown above was caused by the test described in section 2.2.2. See section 3.6 for units of quantities shown.

FIGURE 8

3.3.3 SAMPLE CASE #3: Perform Ephemeris Calculation With Error Analysis. Output All Quantities Possible.

(a) Input cards used: This is a description of the cards used and the reasons for their use, listed in the order in which the cards appear in the input deck (see Figure 9).

(1) ISENT CARD (see Figure 11) with a -1 punched in columns 5 and 6. This card tells the program to bypass the input of the SENSØR and SIGMA cards, since these are not needed for ephemeris calculation.

(2) IDCEPH CARD (see Figure 16) with a -1 punched in columns 5 and and 6. This card directs the program to the ephemeris calculation section.

(3) 7 ELEMENT CARDS (see Figures 17 to 23): These cards contain the initial elements needed for the program. They are standard SPADATS format. An E must appear in column 80 of each card. These standard elements are converted to the $\underline{N}$, $\underline{M}$ elements by the program.

(4) IØPT, ICAL CARD (see Figure 31): The 6 in column 6 is the IØPT indicator. Since it is 6, it will cause the program to print out all output possible (see Section 3.2.4). The 2 in column 12 is the ICAL indicator. This directs the program to the ephemeris calculation with error analysis (see Section 3.2.3).

(5) 2 TERMS CARDS (see Figures 25 and 26, and Section 3.2.9): These cards tell the program which of the 89 General Perturbations terms to use in the nominal case for the error analysis. In this case all 89 terms are used.

(6) Input for XYZ Subroutine (see Section 3.2): The first six quantities are the epoch time and are punched as shown in Figure 32.

The 0.5 in columns 38, 39 and 40 is AGØM. This is the quantity $\frac{A\gamma}{m}$ and since it is less than 1, it causes the program to omit the calculation of the radiation pressure perturbations.

The 0.0 in columns 44, 45 and 46 is DTERM. Since it is zero, it tells the program to calculate DTERM, the period decay acceleration.

The 1000.0 in columns 67 through 72 is TEND. This is the time interval to be covered in the ephemeris calculation. Since TEND is equal to DELTAT, the program will calculate the error analysis for only two times, t_o and t_o +1000 minutes (Note: no more than 300 time increments may be used in the error analysis section, i.e., $\frac{\text{TEND}}{\text{DELTAT}}$ must be less than 300.)

(7) 2 NTERMS (I) CARDS (see Figures 25 and 26, and Section 3.2.9): These cards tell the program which of the 89 General Perturbations terms to use in the omitted term case for the error analysis. In this case, only terms 1, 6, 42, 49, 50 and 88 are included.

(8) IBACK1 CARD (see Figure 33 and Section 3.2.5) with a 4 punched in column 6. This card tells the program to end after the error analysis.

(9) END DATA CARD (see Figure 34) An END DATA CARD (as in Sample Case #1) must appear after the last card in the data deck.

(b) Output from Sample Case #3

The program prints out the quantities described in Section 3.1.2c, as shown by Figure 10. Only the output for $t = t_o + 1000$ minutes is shown.

3.4 Input Card Format

This section describes the input cards used in this program. Examples of the use and contents of these cards may be found in Section 3.3.

CARDS	COLUMNS 1	13	25	37	49	61 … 72	80
(1) ISENT	-1						
(2) IDCEPH	-1						
(3) ELEMENT CARD #1	666001 1 SAM	PLE		0.001	30.0		E
ELEMENT CARD #2	666001 21959	26	8.149861		60.0		E
ELEMENT CARD #3	666001 3		0.0	30	.01.0224942 1	7	E
ELEMENT CARD #4	666001 4-0.1	5E-4					E
ELEMENT CARD #5	666001 5						E
ELEMENT CARD #6	666001 6						E
ELEMENT CARD #7	666001 7						E
(4) IOPT, ICAL	6 2						
(5) TERMS CARD #1	111111111111	111111111111	111111111111	111111111111	111111111111	111111111111	11111111
TERMS CARD #2	1111111111						
(6) (SEE DESCRIPT.)	3 04	04 11	43 28.00	0.5 0.0	1000.0	1000.0	
(7) TERMS CARD #1	1 1			1	11		
TERMS CARD #2	1						
(8) IBACK1	4						
(9) END DATA	END DATA						

R02163B

FIGURE 9. INPUT CARD DATA FOR SAMPLE CASE NO. 3

```
                    EXPERIMENTAL GENERAL PERTURBATIONS DIFFERENTIAL CORRECTION PROGRAM
                           (EPHEMERIS OUTPUT UNDER OUTPUT OPTION NO. 6)

SATELLITE NO. 666  SATELLITE NAME: SAMPLE     ELEMENT SET NO.  0              TIME OF EPOCH    268.1498599

     TIME
Y MM DD HH MM SS.SS
3  4  4 11 43 28.00

NOMINAL CASE (I.E., ALL TERMS INCLUDED)

      T             X              Y              Z             XDOT           YDOT           ZDOT
     MIN.          E.R.           E.R.           E.R.         E.R./KEMIN     E.R./KEMIN     E.R./KEMIN
 1.0000000+003 -4.5917556-001 -7.7432436-001 -4.7016098-001  8.0383948-001 -4.1073781-001 -1.8499131-001

      T            XLAT           ELONG           H
     MIN.          DEG.           DEG.           KM.
 1.0000000+003 -2.7734669+001 -6.0515874+001  1.0410859+002

TERMS  (1)...(89)
   ( 1)           ( 2)           ( 3)           ( 4)           ( 5)           ( 6)           ( 7)           ( 8)
-9.6351684-005  2.4889470-008 -2.2105271-007 -3.2667553-014 -3.3157907-013  1.5297884-004  7.1625600-008  2.0207257-007

   ( 9)           (10)           (11)           (12)           (13)           (14)           (15)           (16)
 7.9371777-014  2.9513245-013  0.             0.             0.             0.             0.             0.

   (17)           (18)           (19)           (20)           (21)           (22)           (23)           (24)
0.             0.             0.             0.             0.             0.             0.             0.

   (25)           (26)           (27)           (28)           (29)           (30)           (31)           (32)
0.             0.             0.             0.             0.             0.             0.             0.

   (33)           (34)           (35)           (36)           (37)           (38)           (39)           (40)
0.             0.             0.             0.             0.             0.            -2.1330407-008  5.1800821-014

   (41)           (42)           (43)           (44)           (45)           (46)           (47)           (48)
 7.9765461-015  5.6627160-005  9.6515670-008  4.6704224-014 -1.8980848-008 -3.6446022-014  0.             0.

   (49)           (50)           (51)           (52)           (53)           (54)           (55)           (56)
0.             0.             0.             0.             0.             0.             0.            -3.8031920-004

   (57)           (58)           (59)           (60)           (61)           (62)           (63)           (64)
 1.3923581-006  5.6801263-015 -1.0522308-011  9.8424295-008  1.0262346-007  1.7562386-007 -4.7747178-005  8.7792641-008

   (65)           (66)           (67)           (68)           (69)           (70)           (71)           (72)
 1.1295229-014 -3.6767554-011 -1.6002037-015  0.             2.7918905-008 -9.0918462-005 -2.2860531-011  6.7903952-008

   (73)           (74)           (75)           (76)           (77)           (78)           (79)           (80)
-1.0536716-011  4.6377381-005  4.8844800-004 -1.4805154-004 -3.8206419-011  1.5450743-007  3.7157527-008 -3.1621316-011

   (81)           (82)           (83)           (84)           (85)           (86)           (87)           (88)


 4.7598615-004 -8.7695819-007 -1.4612918-007 -1.1849937-007 -2.4440431-004  1.7975421-007  2.5037480-008  5.6878916-004

   (89)
-2.8012287-013


   RDELT1         RDELT3         ABSDR8         DELTH
    KM.            KM.            KM.            KM.
 2.0446348+000  1.5227111-001  2.0502962+000 -3.0285926-001
```

This is the output obtained from data select nine from sample case number 3. The zero value of the terms shown above was caused by the test described in section 2.2.2.

FIGURE 10

OMITTED TERM CASE

T MIN.	X E.R.	Y E.R.	Z E.R.	XDOT E.R./KEMIN	YDOT E.R./KEMIN	ZDOT E.R./KEMIN
1.0000000+003	-4.5927906-001	-7.7417723-001	-4.7040785-001	8.8339742-001	-4.1074195-001	-1.8497583-001

T MIN.	XLAT DEG.	ELONG DEG.	H KM.
1.0000000+003	-2.7749040+001	-6.8526382+001	1.0450549+002

TERMS (1)...(89)

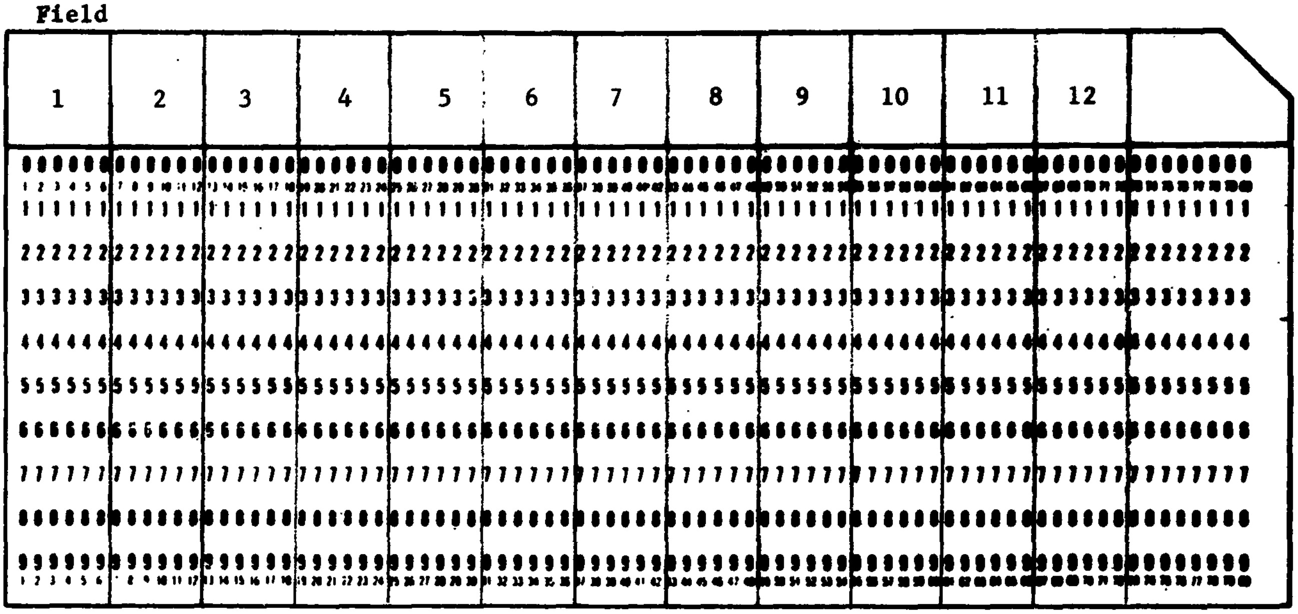
(1) -9.6351684-005	(2) 0.	(3) 0.	(4) 0.	(5) 0.	(6) 1.5297884-004	(7) 0.	(8) 0.
(9) 0.	(10) 0.	(11) 0.	(12) 0.	(13) 0.	(14) 0.	(15) 0.	(16) 0.
(17) 0.	(18) 0.	(19) 0.	(20) 0.	(21) 0.	(22) 0.	(23) 0.	(24) 0.
(25) 0.	(26) 0.	(27) 0.	(28) 0.	(29) 0.	(30) 0.	(31) 0.	(32) 0.
(33) 0.	(34) 0.	(35) 0.	(36) 0.	(37) 0.	(38) 0.	(39) 0.	(40) 0.
(41) 0.	(42) 5.6627160-005	(43) 0.	(44) 0.	(45) 0.	(46) 0.	(47) 0.	(48) 0.
(49) 0.	(50) 0.	(51) 0.	(52) 0.	(53) 0.	(54) 0.	(55) 0.	(56) 0.
(57) 0.	(58) 0.	(59) 0.	(60) 0.	(61) 0.	(62) 0.	(63) 0.	(64) 0.
(65) 0.	(66) 0.	(67) 0.	(68) 0.	(69) 0.	(70) 0.	(71) 0.	(72) 0.
(73) 0.	(74) 0.	(75) 0.	(76) 0.	(77) 0.	(78) 0.	(79) 0.	(80) 0.
(81) 0.	(82) 0.	(83) 0.	(84) 0.	(85) 0.	(86) 0.	(87) 0.	(88) 5.6878916-004
(89) 0.							

RDELT1 KM.	RDELT3 KM.	ABSDRB KM.	DELRE RAD.	DELUE RAD.	DELIE RAD.	DELNOE RAD.
1.8053991+000	6.8484425-001	1.9544793+000	-3.0250887-001	-1.3562245-004	-2.4419863-004	2.7868096-004

See section 3.6 for units and meaning of quantities shown.

FIGURE 10 (Cont.)

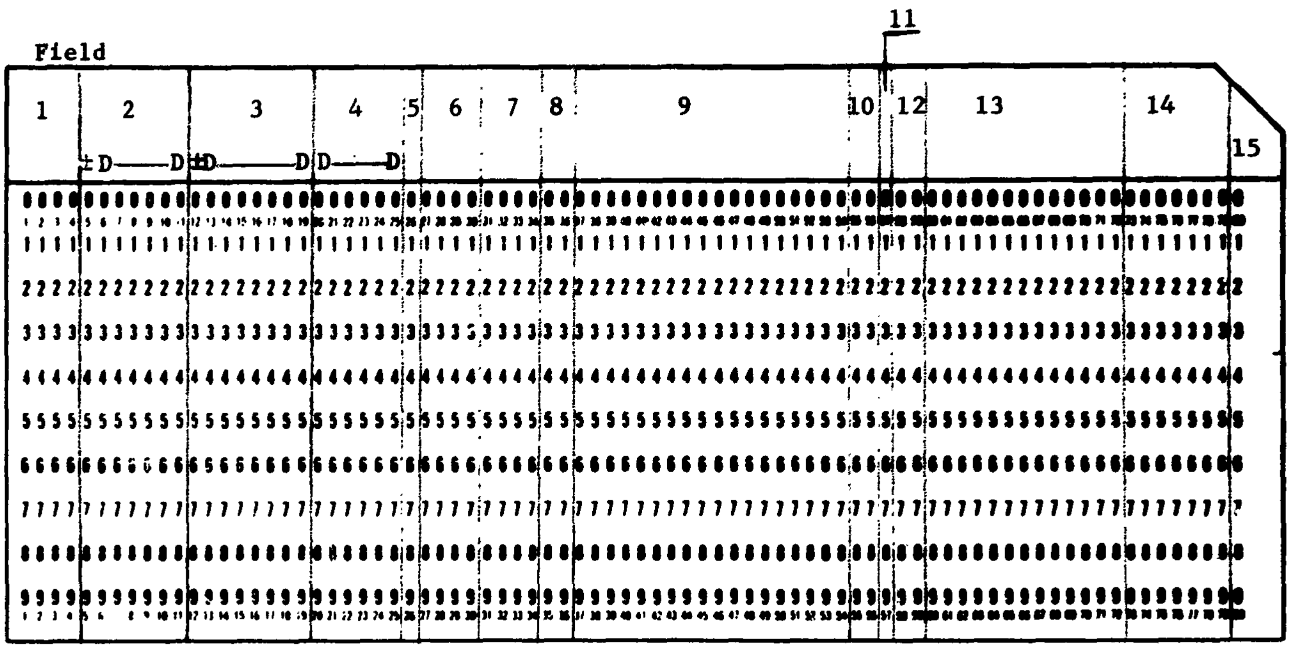
Field



Field	Columns	Description*
1	1 - 6	Input specification (right adjusted integer) "+1" means input sensor data with cards and then input SIGMA cards. "-1" means bypass SIGMA card input and input sensor data from the SEAI (sensor, element, acquisition, information) tape for D.C. option or input no sensor data for ephemeris option "0" (or Blank card) means input SIGMA cards and input sensor data from tape
2 - 12	7 - 72	Not used as this time.

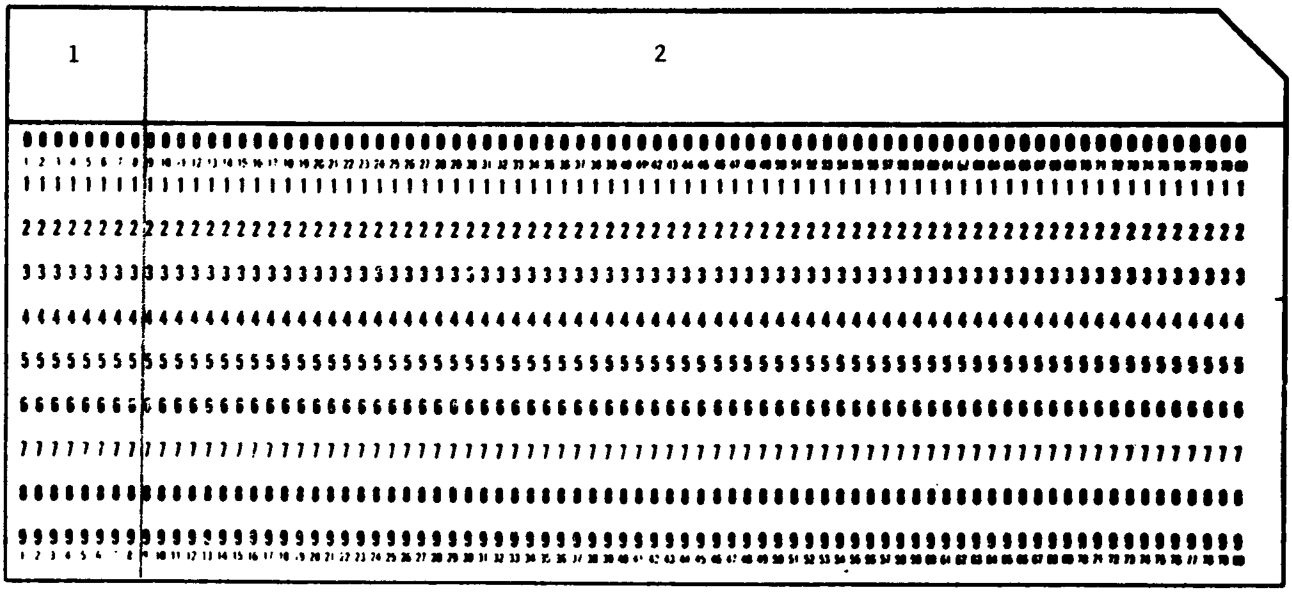
*Note: See summary of program options.

FIGURE 11 ISENT CARD



Field	Columns	Description
1	1 - 4	Sensor Number
2	5 - 11	φ° (+N) - latitude (decimal assumed between cols. 7-8)
3	12 - 19	λ° (+W) - longitude (" " " " 15 - 16)
4	20 - 25	H (meters) - altitude (" " after col. 25)
5	26	Classification
6	27 - 30	Sensor Type
7	31 - 34	Previous Sensor Number
8	35 - 36	Number within Sensor Complex
9	37 - 54	Name
10	55 - 56	Equipment type
11	57	Continent
12	58 - 59	Country or State
13	60 - 72	Comments
14	73 - 79	Not used
15	80	Card type = S (Punch 0, 2)

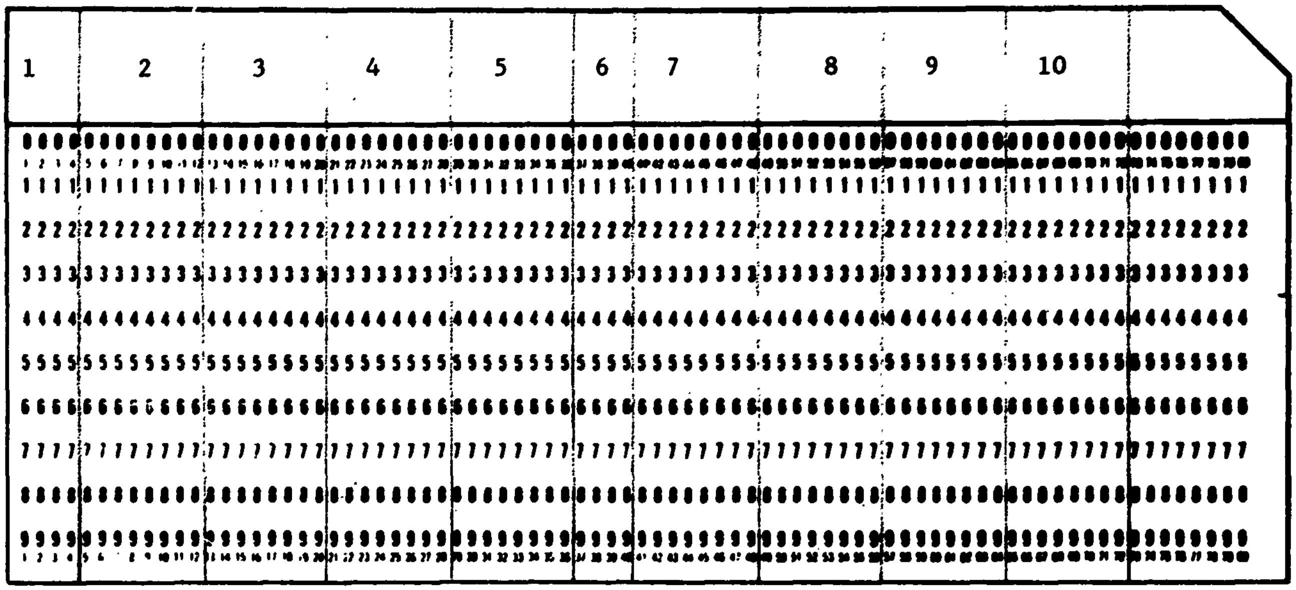
FIGURE 12 SENSOR CARD



Field	Columns	Description
1	1 - 7	ENDSENS is punched in columns 1 through 7. This card must appear after the last SENSOR card.
2	8 - 80	Not used as this time.

FIGURE 13 ENDSENS CARD

Field



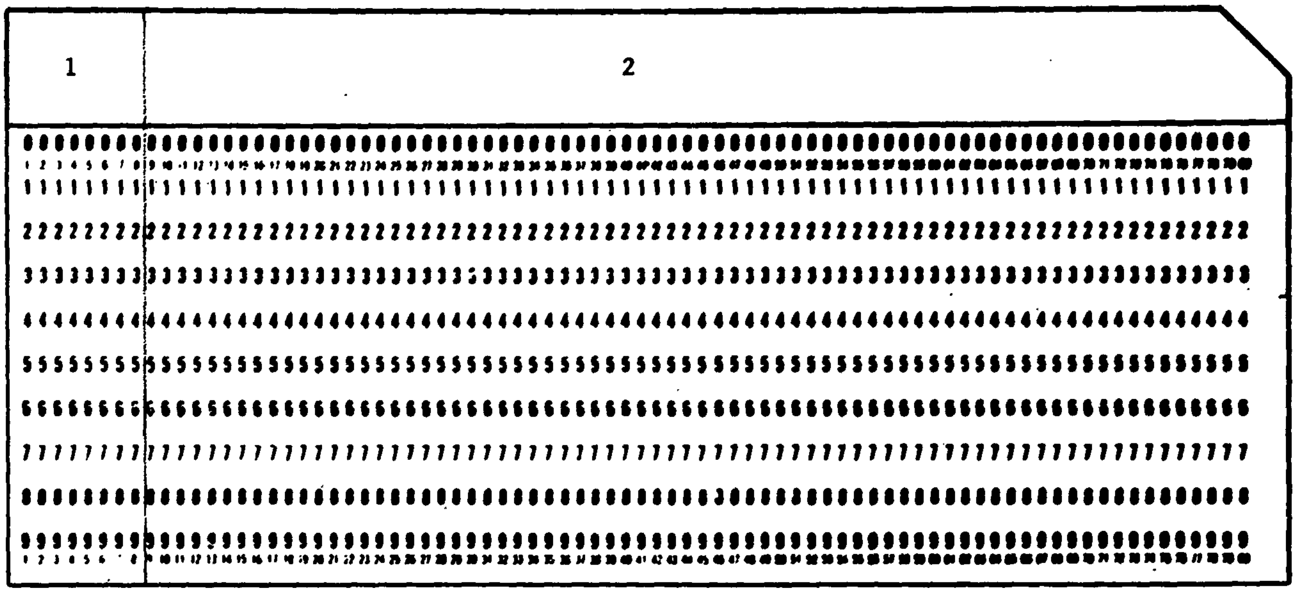
Field	Columns	Description
1	1 - 4	Sensor number (right adjusted integer)
2	5 - 12	σ_1: Range standard deviation (s.d.) (meters)
3	13 - 20	σ_2: Azimuth or right ascension (s.d.) (deg.)
4	21 - 28	σ_3: Elevation or declination (s.d.)(deg.)
5	29 - 36	σ_4: Range-rate (s.d.) (meters/sec.)
6	37 - 40	Next sensor number
7 - 10	41 - 72	Four eight-column fields corresponding to fields 2-5 but used with sensor indicated in field 6.

Notes:

1. All entries are floating point numbers.
2. See Differential Correction Control card (Fig. 27) for SIGMA card options.

FIGURE 14 SIGMA CARD

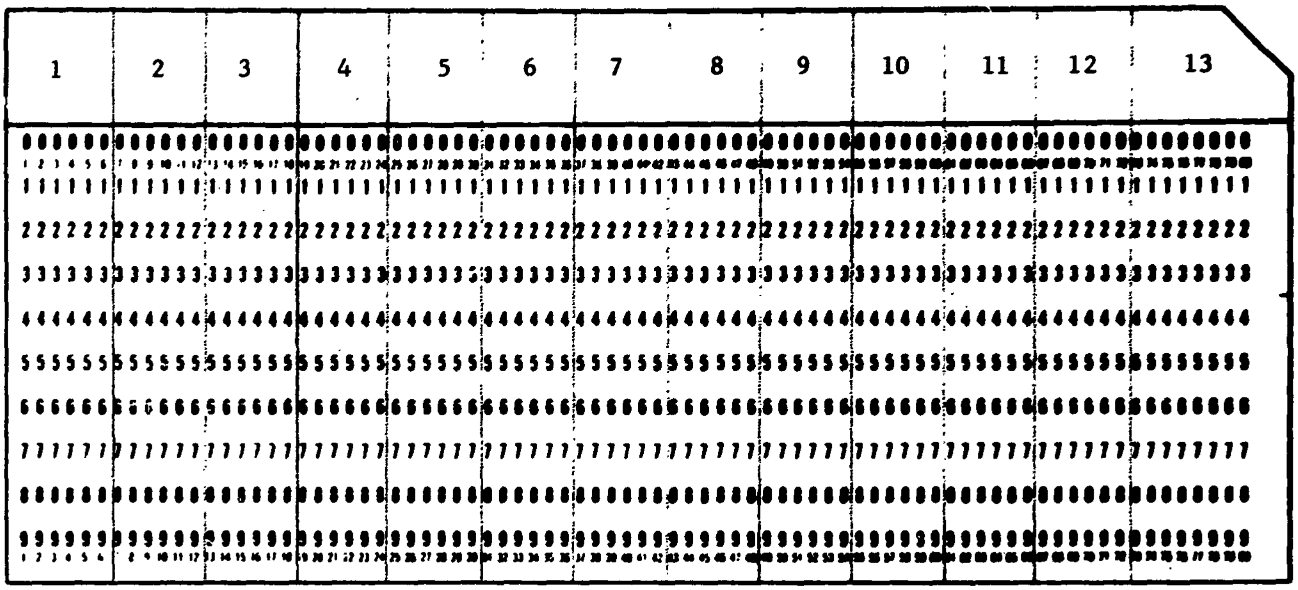
Field



Field	Columns	Description
1	1 - 8	ENDSIGMA is punched in columns 1 through 8. This card must appear after the last SIGMA card or after the ENDSENS card if no SIGMA cards are used. It is not used when ISENT = -1 (See summary of program options, Sec. 3.2)
2	9 - 80	Not used at this time.

FIGURE 15 ENDSIGMA CARD

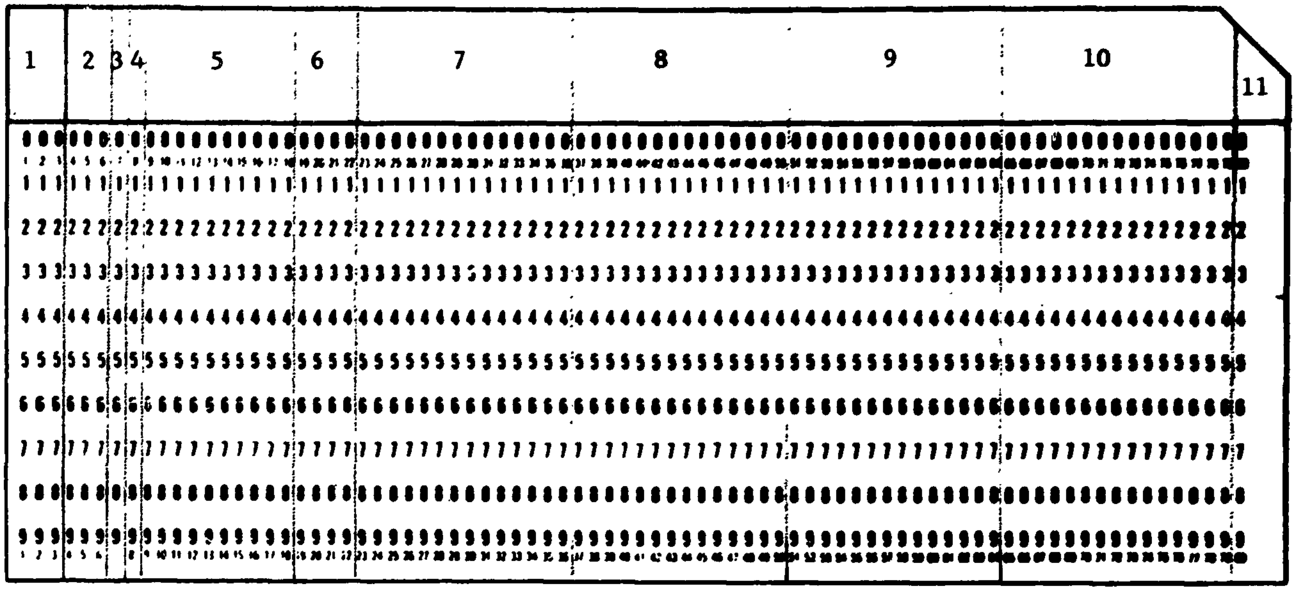
Field



Field	Columns	Description
1	1 - 6	Program option specification (right adjusted integer) "+1" means go to differential correction subroutine "-1" means go to ephemeris calculation subroutine "0" means end program When using the D.C. portion of the program, an IDCEPH = 0 card is used as the last input card in order to end the program.
2 - 13	7 - 80	Not used as this time.

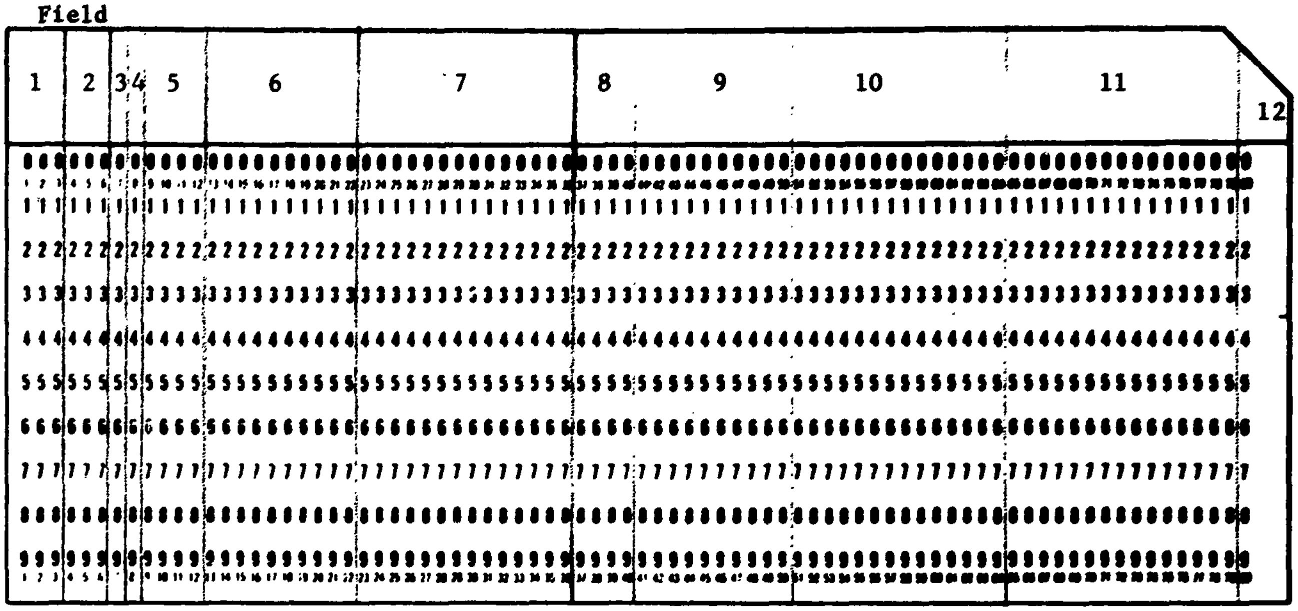
FIGURE 16 IDCEPH CARD

Field



Field	Columns	Description
1	1 - 3	Satellite number - justified right
2	4 - 6	Element set number - justified right
3	7	Not used
4	8	Card Number (Card # = 1)
5	9 - 18	Satellite name for Element File Update
6	19 - 22	Not used
7	23 - 36	N_o - Epoch revolution
8	37 - 50	e - Eccentricity
9	51 - 64	i - Inclination (degrees) (i = 0 may <u>not</u> be used in this program)
10	65 - 79	Not used
11	80	Card type E - Nodal Elements

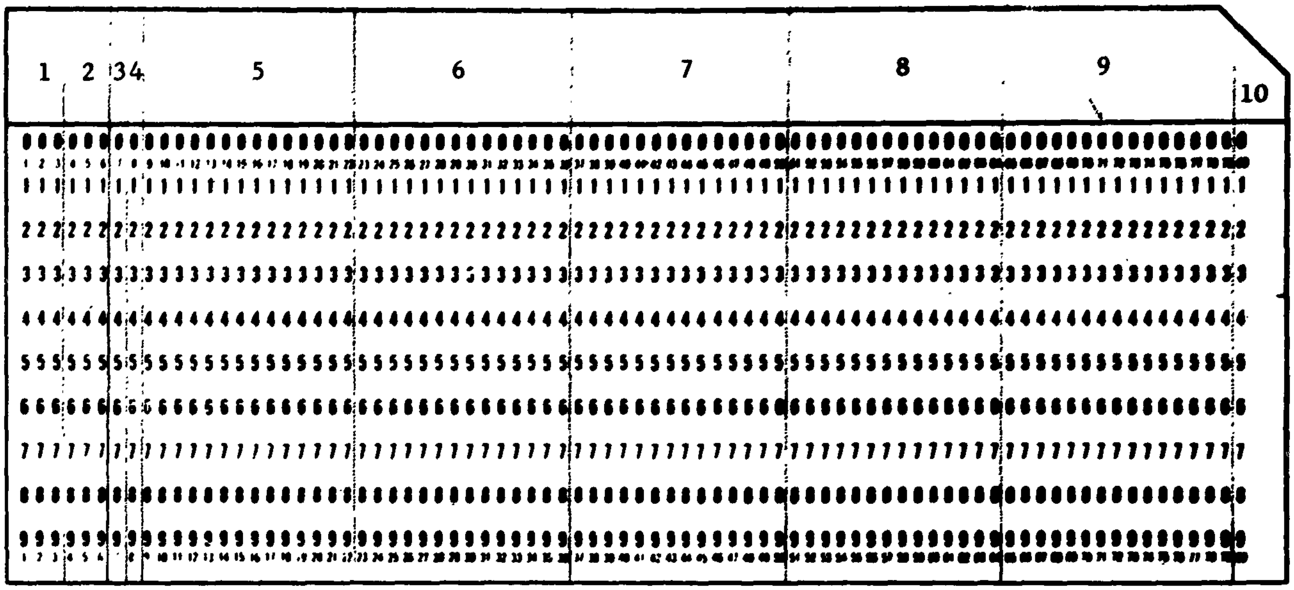
FIGURE 17 ELEMENT CARD 1



Field	Columns	Description
1	1 - 3	Satellite Number - justified right
2	4 - 6	Element set number
3	7	Not used
4	8	Card number (Card # = 2)
5	9 - 12	Year of T_o
6	13 - 22	Not used
7	23 - 36	T_o - Time of Epoch (day and fraction of days in year)
8	37 - 40	Not used
9	41 - 50	Not used
10	51 - 64	L_o - Mean Longitude - degrees
11	65 - 79	Not used
12	80	Card type E = Nodal Elements

FIGURE 18 ELEMENT CARD 2

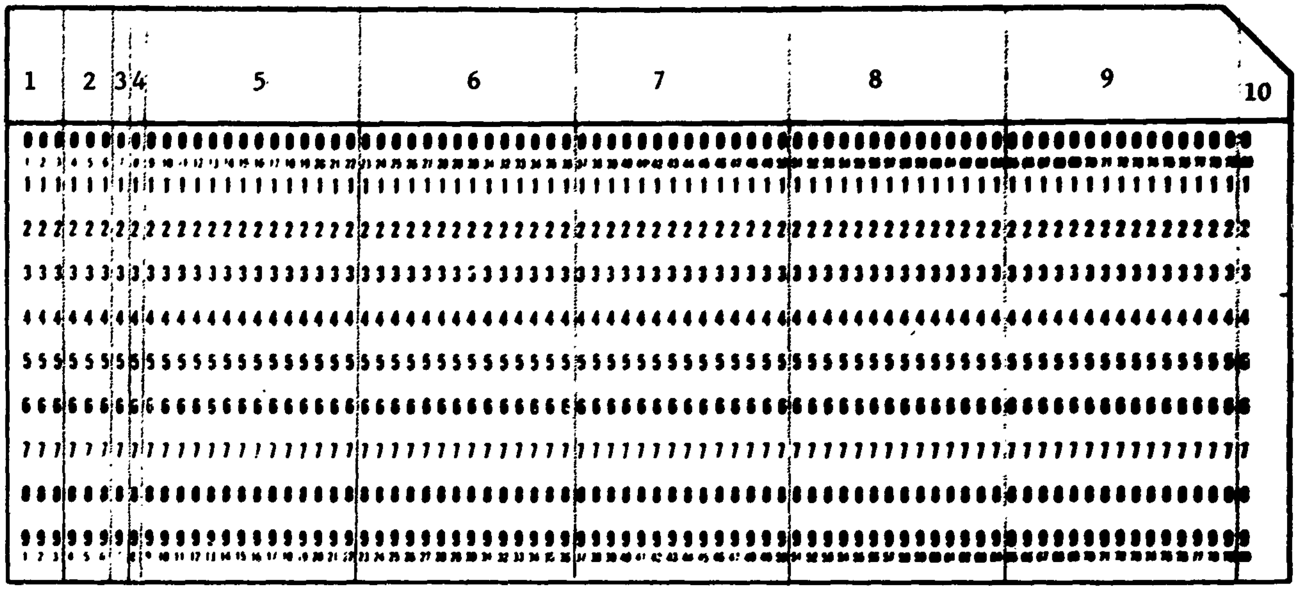
Field



Field	Columns	Description
1	1 - 3	Satellite number - justified right
2	4 - 6	Element set number - justified right
3	7	Not used
4	8	Card number (Card # = 3)
5	9 - 22	P_a - Anomalistic Period at Epoch - days/rev.
6	23 - 36	Ω_o - Right ascension of ascending node - degrees
7	37 - 50	ω_o - Argument of perigee - degrees
8	51 - 64	q_o - Perigee-distance-earth radii
9	65 - 79	Not used
10	80	Card type E = Nodal Elements

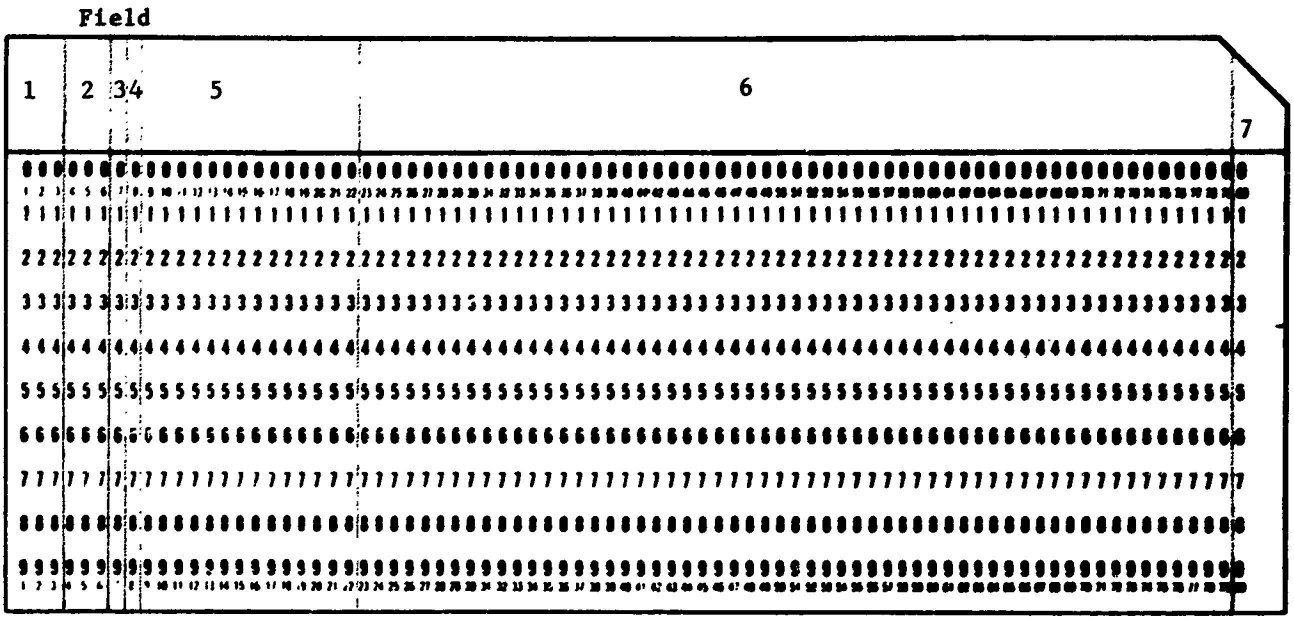
FIGURE 19 ELEMENT CARD 3

Field



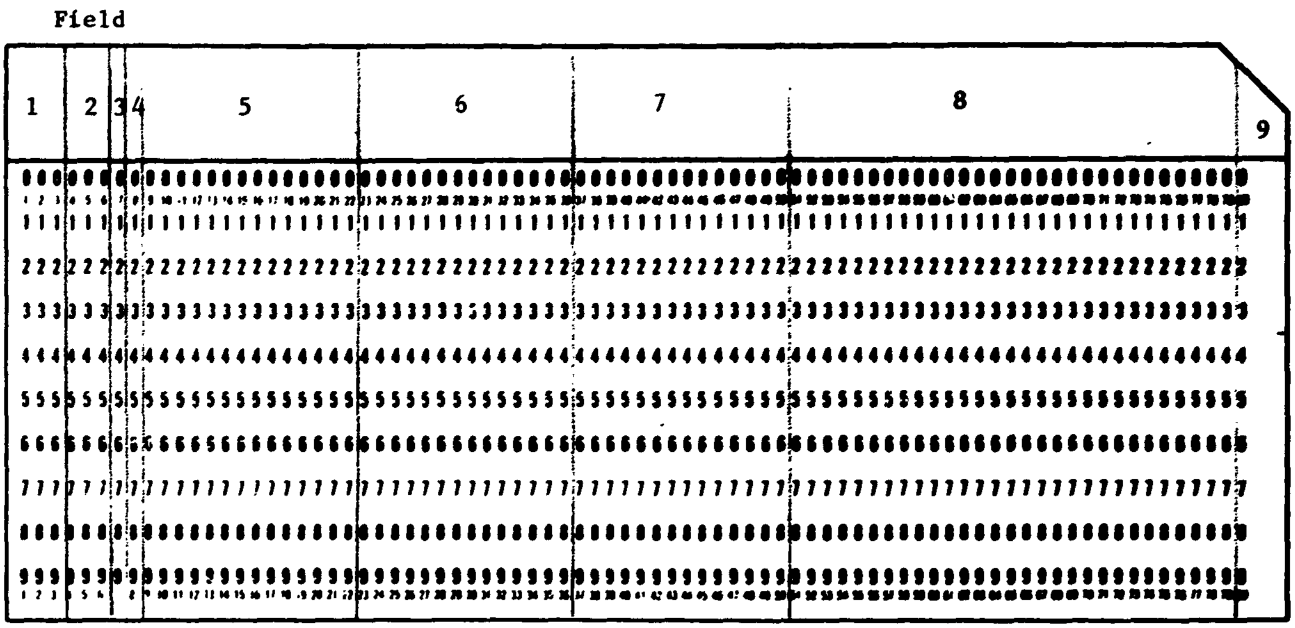
Field	Columns	Description
1	1 - 3	Satellite number - justified right
2	4 - 6	Element set number - justified right
3	7	Not used
4	8	Card number (Card # = 4)
5	9 - 22	c_o - Rate of change of period - days/(rev)^2
6	23 - 36	$\dot{\Omega}_o$ - Time derivative of right ascension of ascending node - degrees/day (Not used by program)
7	37 - 50	$\dot{\omega}_o$ - Time derivative of argument of perigee-degrees/day (Not used by program)
8	51 - 64	Not used
9	65 - 79	Not used
10	80	Card type E = Nodal Elements

FIGURE 20 ELEMENT CARD 4



Field	Columns	Description
1	1 - 3	Satellite Number - justified right
2	4 - 6	Element set number - justified right
3	7	Not used
4	8	Card number (Card # = 5)
5	9 - 22	d - decay acceleration (floating point entry)
6	23 - 79	Not used
7	80	Card type E = Nodal Elements

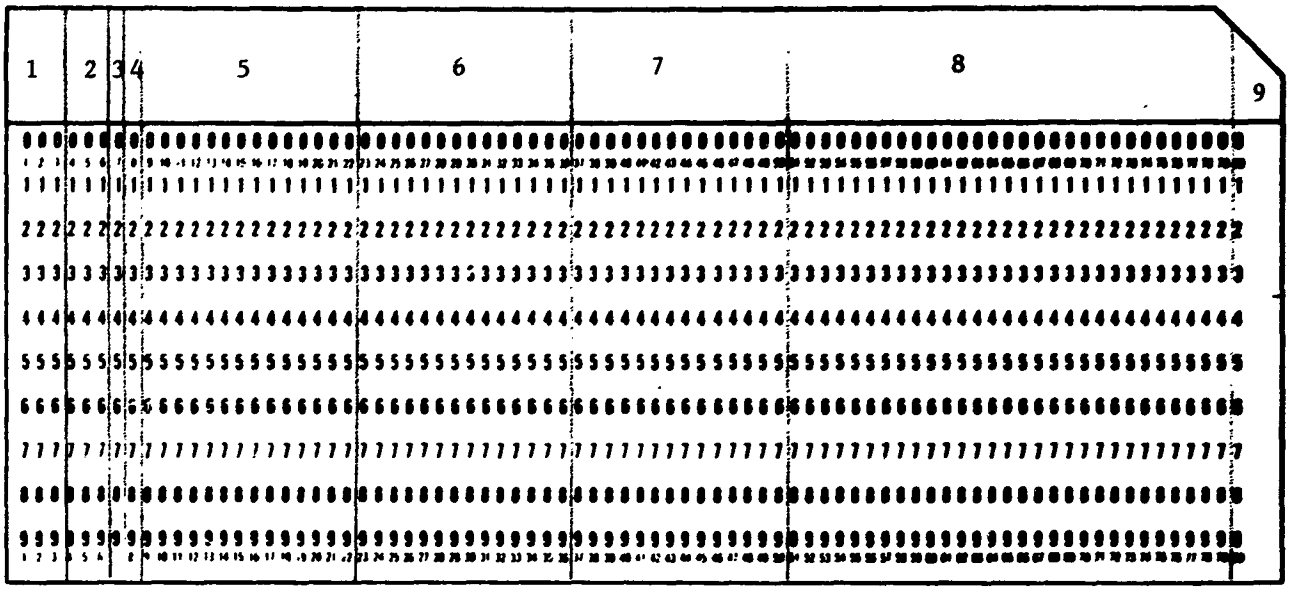
FIGURE 21 ELEMENT CARD 5



Field	Columns	Description
1	1 - 3	Satellite number - justified right
2	4 - 6	Element set number - justified right
3	7	Not used
4	8	Card number (Card # = 6)
5	9 - 22	a - semi-axis major - Earth radii - (Not used by program)
6	23 - 36	P_N - Nodal period - days/rev.
7	37 - 50	c_N - rate of change of nodal period - days/(rev)^2
8	51 - 79	Not used
9	80	Card type E = Nodal elements

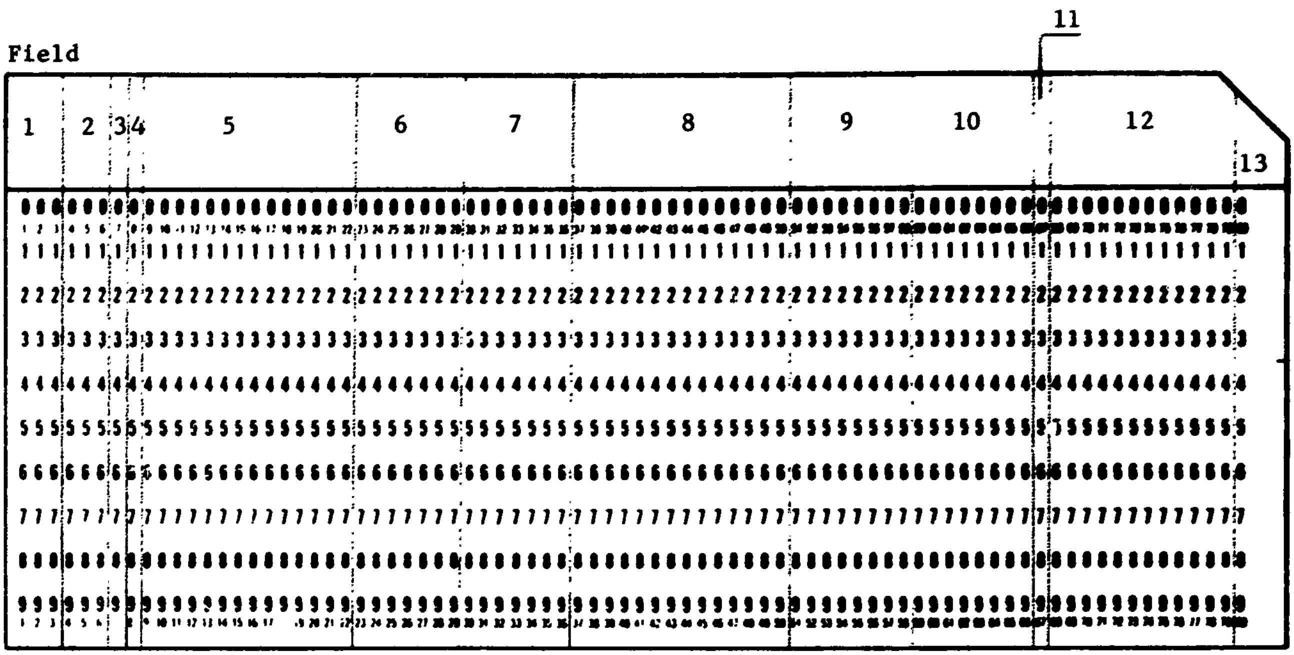
FIGURE 22 ELEMENT CARD 6

Field



Field	Columns	Description
1	1 - 3	Satellite number - justified right
2	4 - 6	Element set number - justified right
3	7	Not used
4	8	Card number (Card # = 6)
5	9 - 22	a - semi-axis major - Earth radii - (Not used by program)
6	23 - 36	P_N - Nodal period - days/rev.
7	37 - 50	c_N - rate of change of nodal period - days/(rev)^2
8	51 - 79	Not used
9	80	Card type E = Nodal elements

FIGURE 22 ELEMENT CARD 6



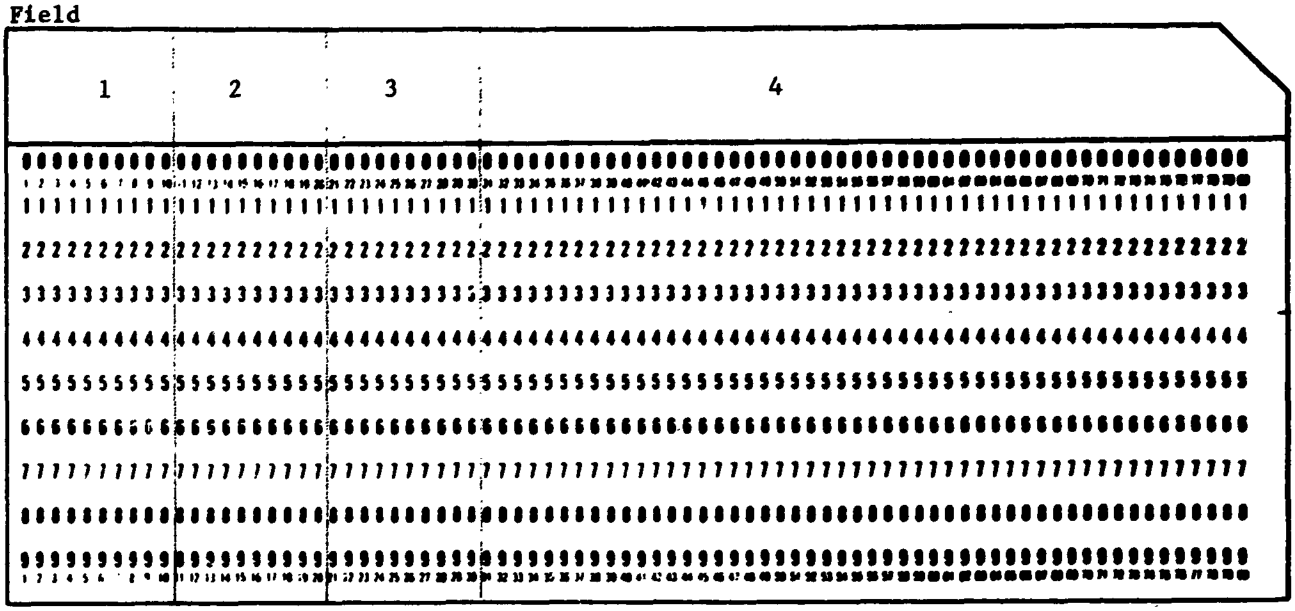
Field	Columns	Description
1	1 - 3	Satellite number
2	4 - 6	Element set number
3	7	Not used
4	8	Card number (Card # = 7)
5	9 - 22	Not used
6	23 - 29	Initial Revolution, decimal may be punched in column 29
7	30 - 36	Fn (Final Revolution); decimal may be punched in column 36
8	37 - 50	Expiration date of Bulletin, in format: YMMDDHHMMSS.SS, decimal punched in column 48
9	51 - 58	RMS, in format XXXXX.XX; decimal punched in column 56

FIGURE 23 ELEMENT CARD 7
(sheet 1 of 2)

Field	Columns	Description
10	59 - 66	Number of observations used in obtaining RMS
11	67	ISTØP
		Blank or 0 = correct the inclination element
		1 = do not correct the inclination
		2 = do not correct the drag parameter
		4 = correct time equation only
12	68 - 79	Not used
13	80	Card type
		E = Nodal Elements

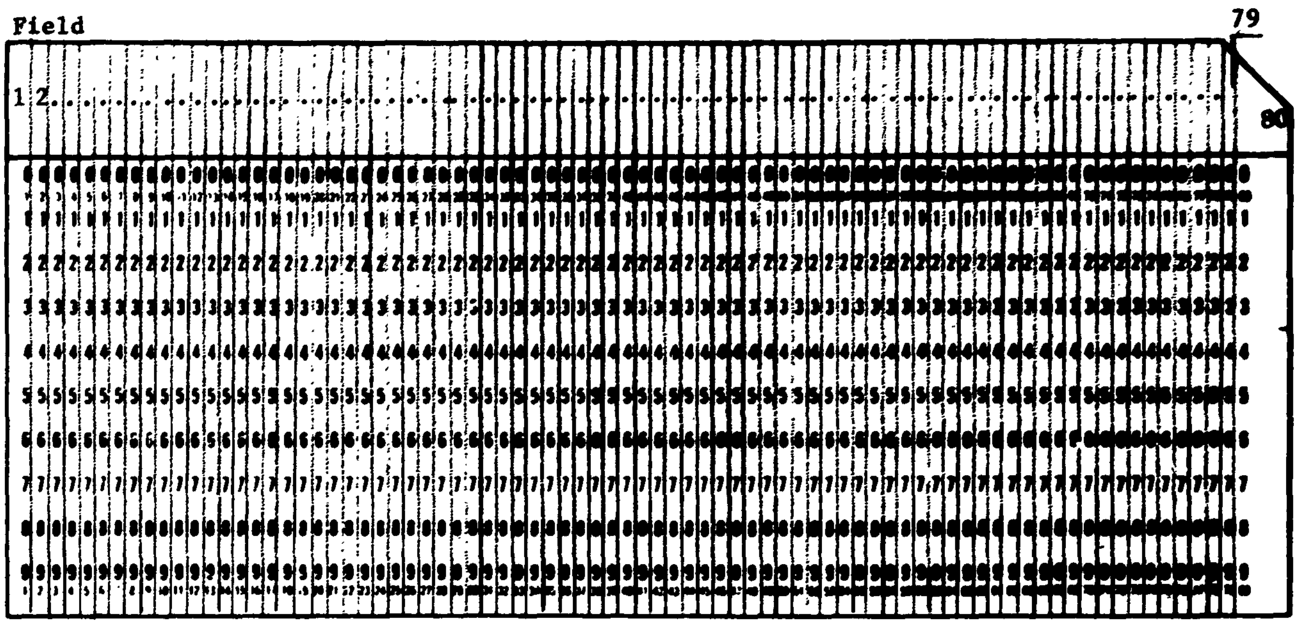
FIGURE 23 ELEMENT CARD 7
(sheet 2 of 2)

Field



Field	Columns	Description
1	1 - 10	DTERM, "d" drag coefficient ($1/\text{min}^2$) (floating point constant or 0.0). If DTERM = 0.0, then "d" is calculated in the program.
2	11 - 20	IØUT, Differential Correction (D.C.) output specification (right adjusted integer). "+1" tells D.C. subroutine to output values calculated in the XYZ subroutine. "0" tells D.C. subroutine to bypass output of values calculated in the XYZ subroutine.
3	21 - 30	AGØM, $\frac{A\gamma}{m}$, used in radiation pressure calculation (floating point constant). If AGØM ≥ 1 the XYZ subroutine calculates radiation pressure effects. If AGØM <1 the XYZ subroutine bypasses the calculation of radiation pressure effects.
4	31 - 80	Not used at this time.

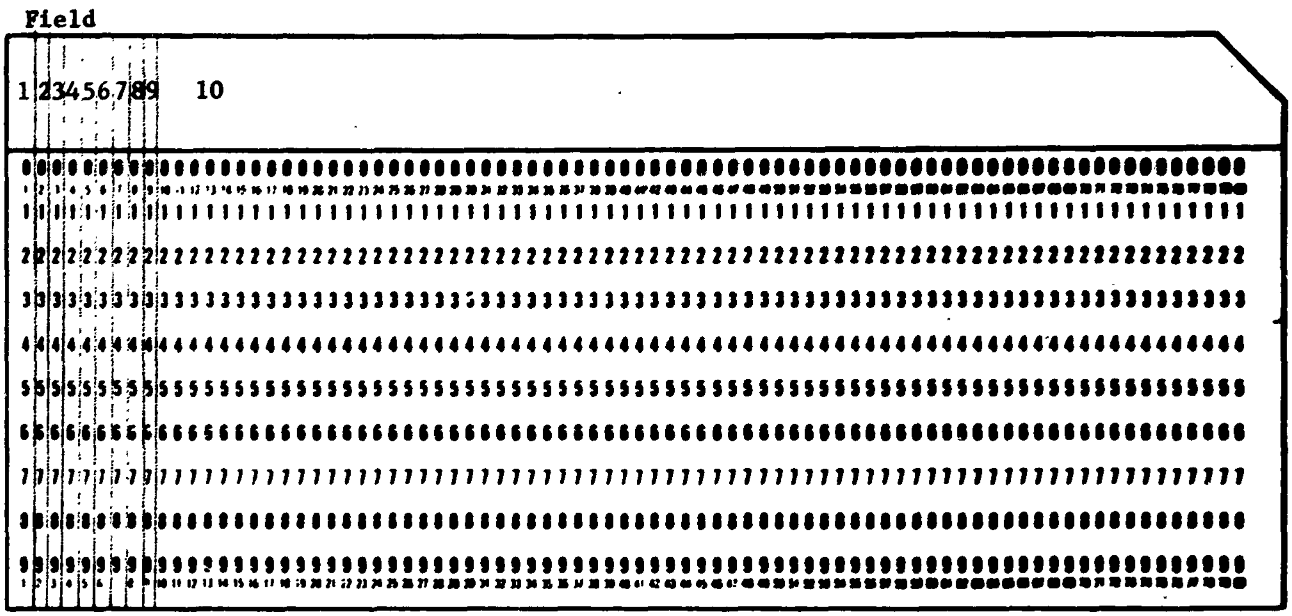
FIGURE 24 AGØM CARD



Field	Columns	Description*
1	1	Fixed point 1 or blank
2	2	Fixed point 1 or blank
3	3	Fixed point 1 or blank
.	.	.
.	.	.
.	.	.
80	80	Fixed point 1 or blank

*Note: Columns 1 through 80 correspond to terms Q(1) through Q(80) in the program. A one punched in any column will cause the corresponding term to be included in the ephemeris calculation. A blank column will cause the corresponding term to be set equal to zero in the ephemeris calculation.

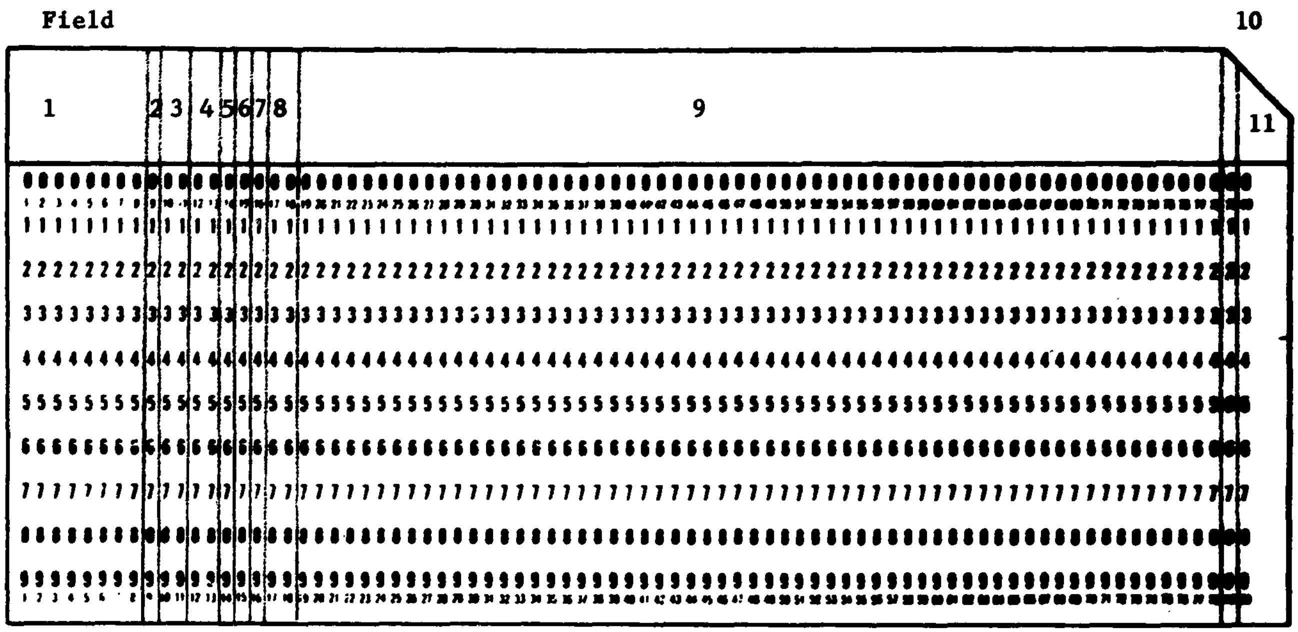
FIGURE 25 TERMS CARD 1



Field	Columns	Description*
1	1	Fixed point 1 or blank.
2	2	Fixed point 1 or blank.
3	3	Fixed point 1 or blank.
4	4	Fixed point 1 or blank.
5	5	Fixed point 1 or blank.
6	6	Fixed point 1 or blank.
7	7	Fixed point 1 or blank.
8	8	Fixed point 1 or blank.
9	9	Fixed point 1 or blank.
10	10 - 80	Not used at this time.

*Note: Columns 1 through 9 correspond to terms Q(81) through Q(89) in the program. See note on previous page for interpretation of 1 or blank entry.

FIGURE 26 TERMS CARD 2



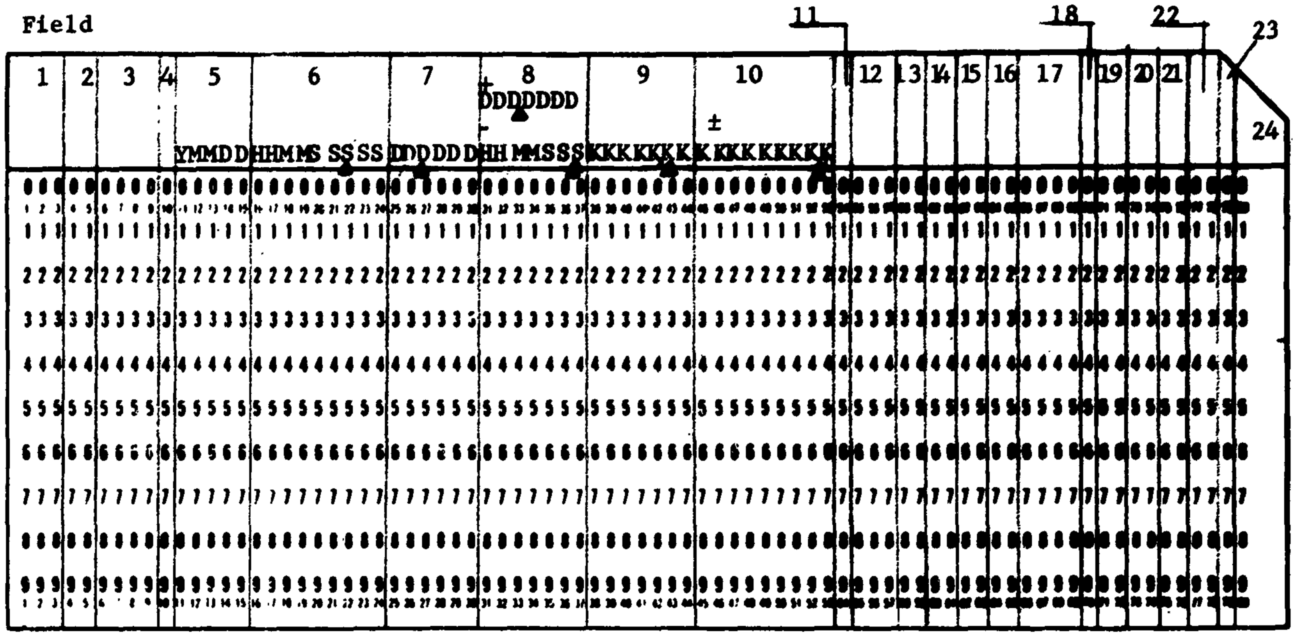
Field	Columns	Description
1	1 - 8	Correction specification. "1" = Correct, "0" = Do not correct the corresponding element. Elements are a, a_{xN}, a_{yN}, U, Ω, i, c", and d in that order. If field not used, program follows nominal SPADATS sequence.*
2	9	Number of times to correct elements specified by Field 1, regardless of convergence.
3	10 - 11	Correction pattern specification. 1 of 10 choices by number. 1 means correct with nominal procedure. Others not used at this time.
4	12 - 13	Convergence criterion override - minimum % change in RMS for convergence (integer $\leq$ 99 and used only when a correction is specified)

*Aeronutronic publication U-1691, revised 1 October 1962, p. 3-65

FIGURE 27 DIFFERENTIAL CORRECTION CONTROL CARD
(sheet 1 of 2)

Field	Columns	Description
5	14	Punched card output type - "0" or blank = standard SPADATS Format. "1" = N M Format.
6	15	Residual output designator. "0" or blank = output first and last pass residuals. "1" = Output all residuals.
7	16	Rejection criterion time factor designator. "0" or "Δ" = Do not used $\frac{t - t_o}{3}$ (time) factor, "1" = use time factor in residual rejection.
8	17 - 18	Absolute maximum for range-rate residuals. Decimal point assumed between col. 17 & 18.
9	19 - 78	Observation rejection overrides for range and angle observations only. Ten fields of six columns each are provided for specifying an RMS multiplier and absolute maximum or an absolute maximum (only) for each iteration specified by field two in the differential correction process. See Section 2.3.9 for further discussion of observation rejection.
10	79	If "0" or blank, then do not use σ's specified on SIGMA cards. If "1," then use σ's on SIGMA cards. If "2", then use σ's on SIGMA cards and mult. factors on obs. cards.
11	80	Not used at this time.

FIGURE 27 DIFFERENTIAL CORRECTION CONTROL CARD
(sheet 2 of 2)



Field	Columns	Description
1	1 - 3	Satellite number. Column 1 contains a minus sign if this is a classified observation; $\underset{o}{+}$ or $\underset{o}{-}$ are not allowed
2	4 - 5	Equipment Type
3	6 - 9	Station Number
4	10	Accuracy or Signal Strength
5	11 - 15	Date
6	16 - 24	Time (Z)
7	25 - 30	Elevation/declination. Column 25 can be overpunched + or - .
8	31 - 37	Azimuth/right ascension. Column 31 can be overpunched + or - . *
9	38 - 44	Slant range (KM)

* A minus overpunch in col. 31 indicates fields 7 and 8 are declination and right ascension, respectively.

FIGURE 28 OBSERVATION CARD
(sheet 1 of 4)

Field	Columns	Description
10	45 - 53	Range rate (KM/sec) with implied decimal point between columns 46 and 47 or: maximum frequency shift (cycles/sec^2) with implied decimal point between columns 52 and 53.
11	54	Code for field 10: 0 or Δ indicates range rate in field 10. 1 indicates max. freq. shift in field 10.
12	55 - 57	At observation time
13	58 - 59	Maximum
14	60 - 61	Minimum
15	62 - 63	Time interval
		(Fields 12–15:) Brightness or, if col. 58 contains a - punch, then: cols. 55-57 = radar cross-section in meters^2 cols. 59-63 = frequency (dec.pt. between 60 and 61) (NOTE: Not used by SPS)
16	64 - 65	Date or line number
17	66 - 69	Message number
18	70	Equinox
19	71 - 72	Multiplying factor for standard deviation of range
20	73 - 74	Multiplying factor for standard deviation of range rate.
21	75 - 76	Multiplying factor for standard deviation of azimuth or right ascension.
22	77 - 78	Multiplying factor for standard deviation of elevation or declination.
23	79	Switch indicator used by manual system.
24	80	Card type (code type = Any numeric between 0 - 9) identifies an Observation card. 0 = Unknown, 1 - 9 coded according to the Association Status as determined in Report Association.

FIGURE 28 OBSERVATION CARD
(sheet 2 of 4)

COLUMN 10 (ACCURACY)

Either accuracy or signal strength may be indicated in column 10, coded according to the following:

> If type, in columns 4 and 5, is 31 or greater, column 10 contains signal strength. If type is 30 or less, column 10 contains accuracy.

Code Figure	Accuracy	Signal Strength
0	Normal observations made under fair conditions.	Signal strength good, reliable measurement.
1	Observations slightly under par due to outside interference (e.g. some clouds, reduced visibility).	Signal fair.
2	Observations only poor due to outside interference.	Signal weak, results poor.
3	Only estimates possible (malfunction of instrument. Too short time of object seeing).	Signal questionable.
4	Doubtful observations, unable to verify either object or instrument behavior. Observations should be considered only as tentative.	

COLUMNS 55 - 63 (CROSS SECTION-FREQUENCY/MAGNITUDE)

The block containing columns 55 through 63 is a dual purpose block where cross section and frequency, or magnitude and time interval are indicated. In order to specify cross section and frequency, a minus is used in column 58. No sign is used in column 58 when this block contains magnitude and time interval.

> Cross section, given in square meters, is listed in columns 55 through 57. To indicate less than one square meter cross section, use appropriate numbers and a minus in column 55, thus in effect, putting a decimal point before column 55. For larger values where three digits would not be sufficient, use a plus in column 55 to represent ten times the indicated value (adding a zero to the value listed).

FIGURE 28 OBSERVATION CARD
(sheet 3 of 4)

Frequency in megacycles, is listed in columns 58 through 63 with the decimal point understood to be located between columns 60 and 61. In rare cases it might be desirable to increase the range of frequency given either side of the decimal point. To do this, use a minus in column 63 to move the point one place to the left, or a plus in column 63 to move the point one place to the right.

COLUMN 70 (EQUINOX)

Column 70 contains year of Equinox as specified by the following:

0 = year of date

1 = 1900

2 = 1925

3 = 1950

4 = 1975

5 = 2000

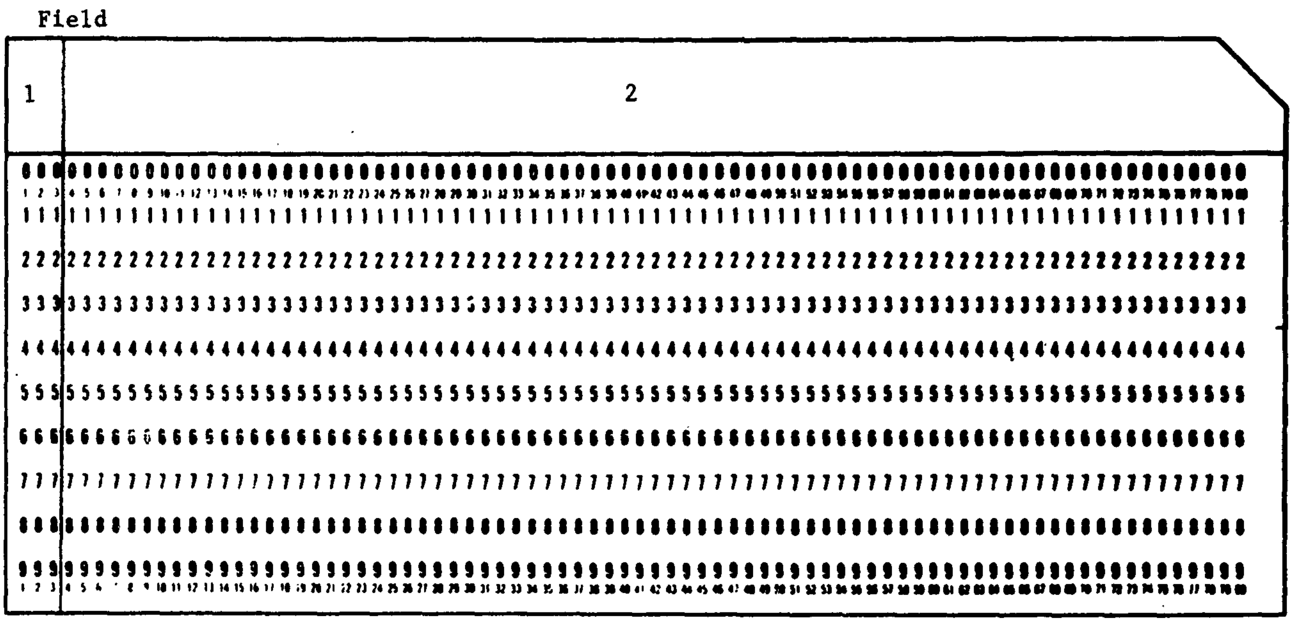
6 = 1850

7 = 1855

8 = 1875

9 = to list actual year, if not provided above, list last two digits of year in columns 71 and 72 and use a minus in column 70 for 18 and a plus in column 70 for 19. Example: Equinox of 1961 would contain "+61" in columns 70, 71, and 72.

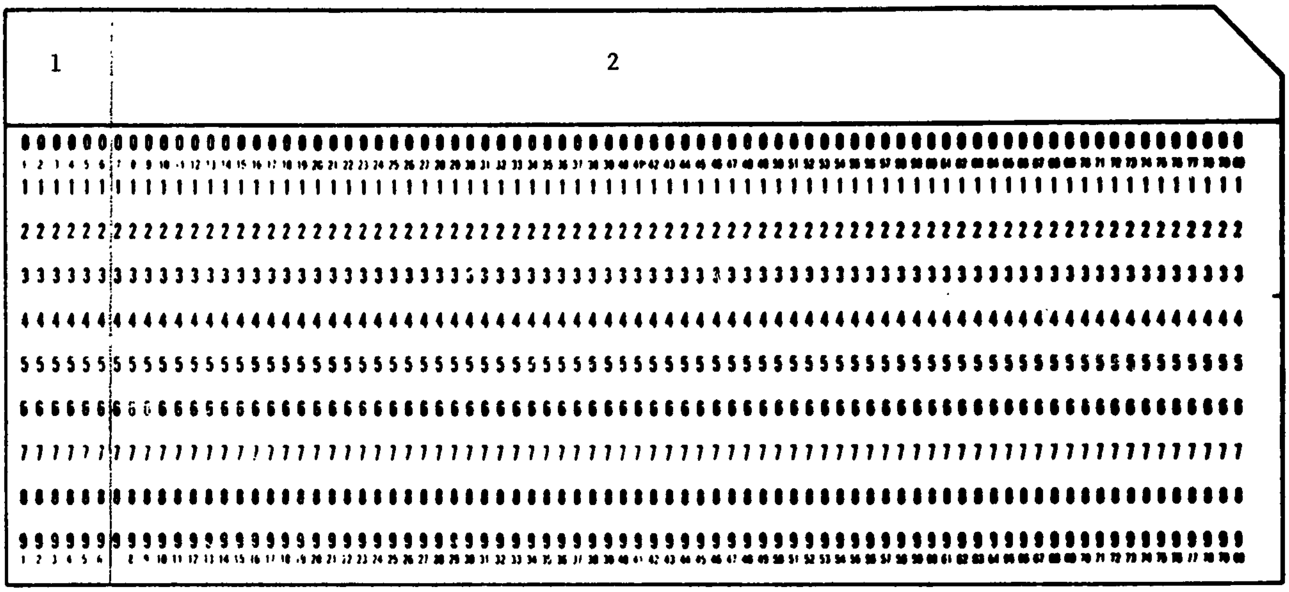
FIGURE 28 OBSERVATION CARD
(sheet 4 of 4)



Field	Columns	Description
1	1 - 3	END punched in columns 1, 2, 3. This card must appear after each batch of observation cards.
2	4 - 80	Not used at this time.

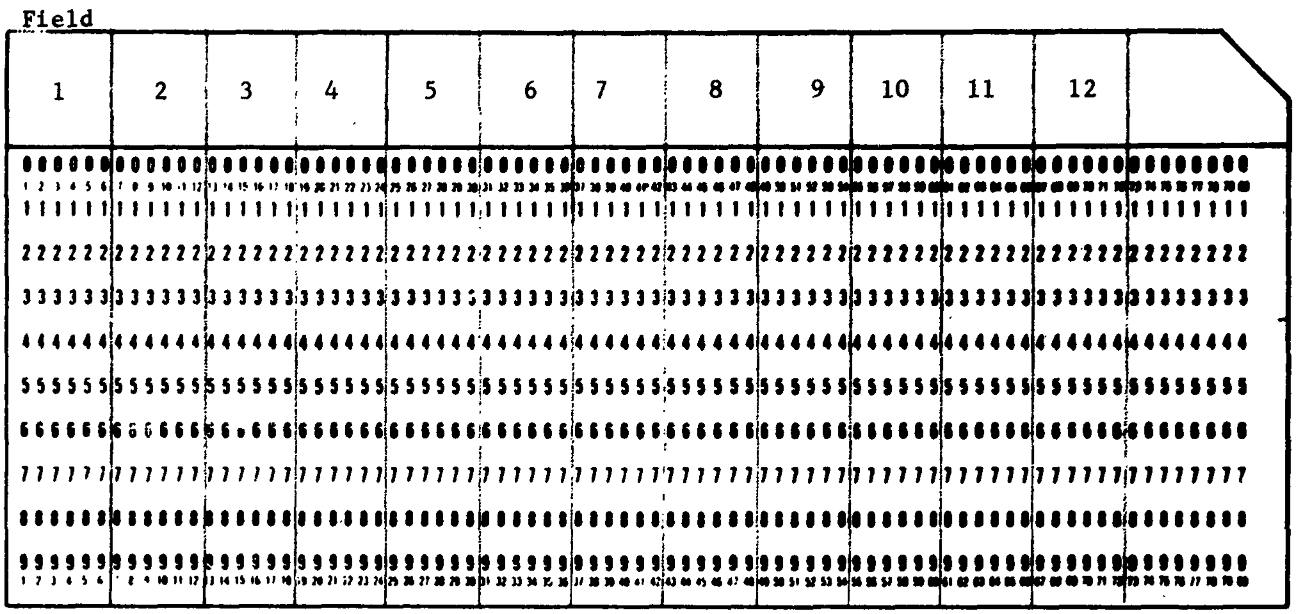
FIGURE 29 END CARD

Field



Field	Columns	Description
1	1 - 6	ENDØBS punched in columns 1 through 6. This card must appear after the END card following the last block of observation cards.
2	7 - 80	Not used at this time.

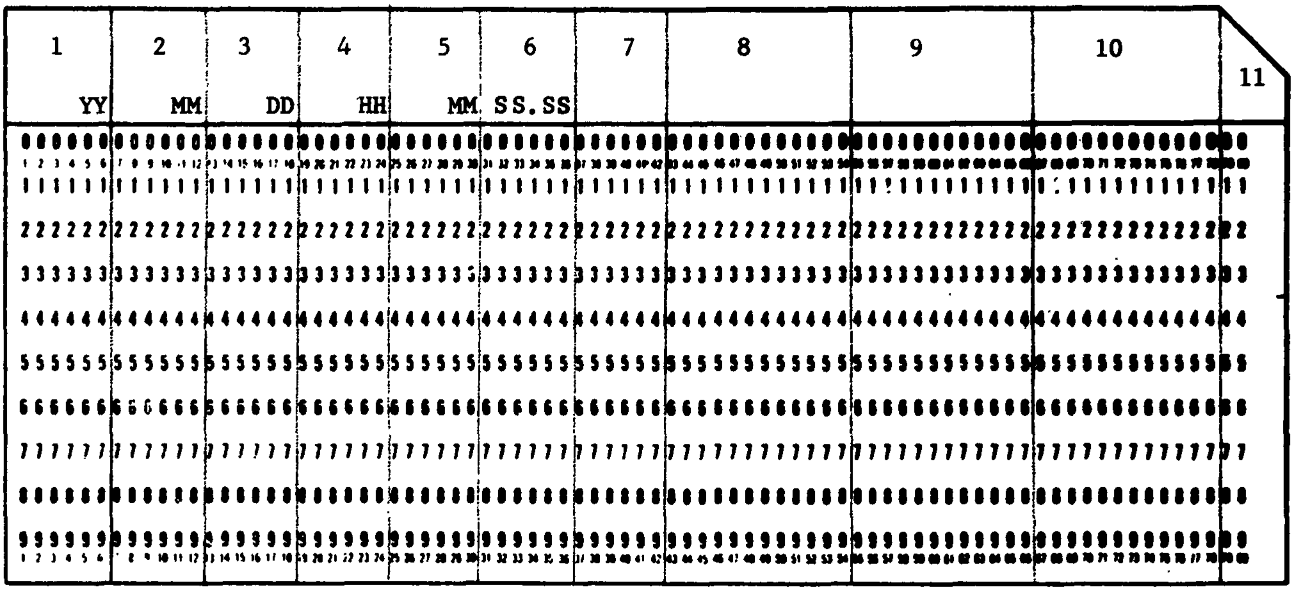
FIGURE 30 ENDØBS CARD



Field	Columns	Description
1	1 - 6	IØPT, XYZ output option selector (right adjusted integer) "1" means output t, x, y, z, $\dot{x}$, $\dot{y}$, $\dot{z}$ "2" means output t, lat., long., height (h) "3" means output t, x, y, z, $\dot{x}$, $\dot{y}$, $\dot{z}$, and lat., long., h "4" means output t, x, y, z, $\dot{x}$, $\dot{y}$, $\dot{z}$ and term values from General Perturbations Section "5" means output t, lat., long., h and term values from General Perturbations Section "6" means output t, x, y, z, $\dot{x}$, $\dot{y}$, $\dot{z}$ and lat., long., h and term values from General Perturbations Section
2	7 - 12	ICAL,error analysis selector (right adjusted integer) "1" means proceed to ephemeris calculation <u>without</u> error analysis "2" means proceed to ephemeris calculation <u>with</u> error analysis
3-12	13 - 72	Not used at this time

FIGURE 31 IØPT, ICAL CARD

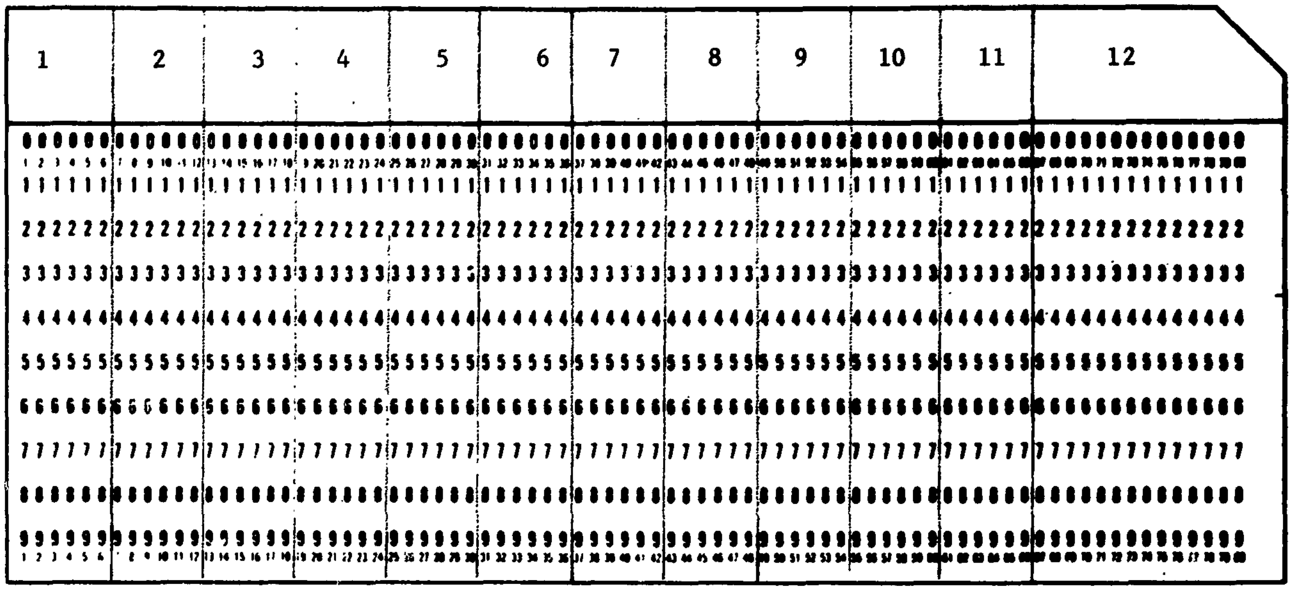
Field



Field	Columns	Description	
1	1 - 6	IYEAR: right adjusted integer	Epoch
2	7 - 12	MØNTH: right adjusted integer	
3	13 - 18	IDAY: right adjusted integer	
4	19 - 24	IHØUR: right adjusted integer	
5	25 - 30	MINUTE: right adjusted integer	
6	31 - 36	SECØND: floating point entry	
7	37 - 42	AGØM, $\frac{A\gamma}{M}$, used in radiation pressure calculation. If AGØM $\geq$ 1 the XYZ subroutine calculates radiation pressure effects. If AGØM $\leq$ 1 the XYZ subroutine bypasses the calculation of radiation pressure effects	
8	43 - 54	DTERM, "d" drag coefficient (floating point constant or 0.0) If DTERM = 0.0, the "d" is calculated in the program.	
9	55 - 66	DELTAT, Δ t, the time increment to use in XYZ subroutine (floating point entry)	
10	67 - 78	TEND, time interval to cover in XYZ subroutine (floating point entry).	
11	79 - 80	Not used at this time	

FIGURE 32 INPUT FOR XYZ SUBROUTINE

Field

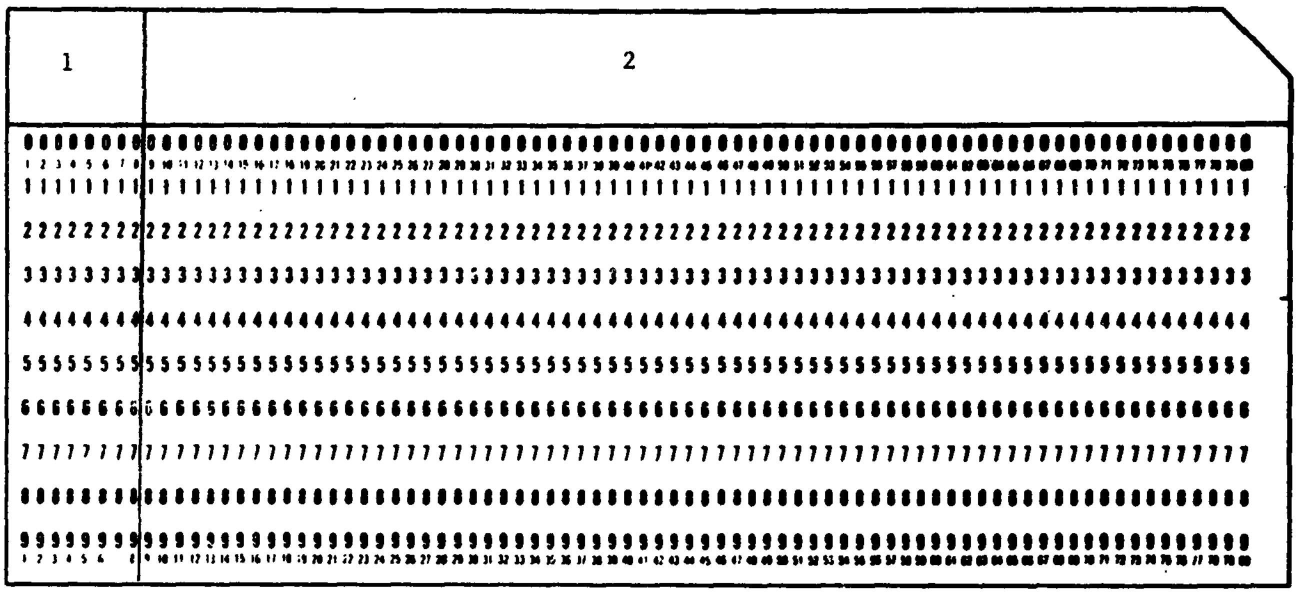


Field	Columns	Description*
1	1 - 6	IBACK1, ephemeris exit option (right adjusted integer) "1" means input new time values and compute another ephemeris using the same terms specified by NTERMS(I) "2" means input new times and terms and compute another ephemeris "3" means input a new value for IDCEPH to determine whether to end program or start another case with new elements "4" means end program
2 - 12	7 - 72	Not used at this time.

*Note: See also Section 3.2

FIGURE 33 IBACK1 CARD

Field



Field	Columns	Description
1	1 - 8	END punched in columns 1-3 and DATA punched in columns 5-8. This card is used at the end of the data deck to tell the computer that all data for this job has been read in.
2	9 - 80	Not used at this time.

FIGURE 34 END DATA CARD

3.5 Flow Charts

The following pages contain the flow charts of the main program and of the more significant subroutines.

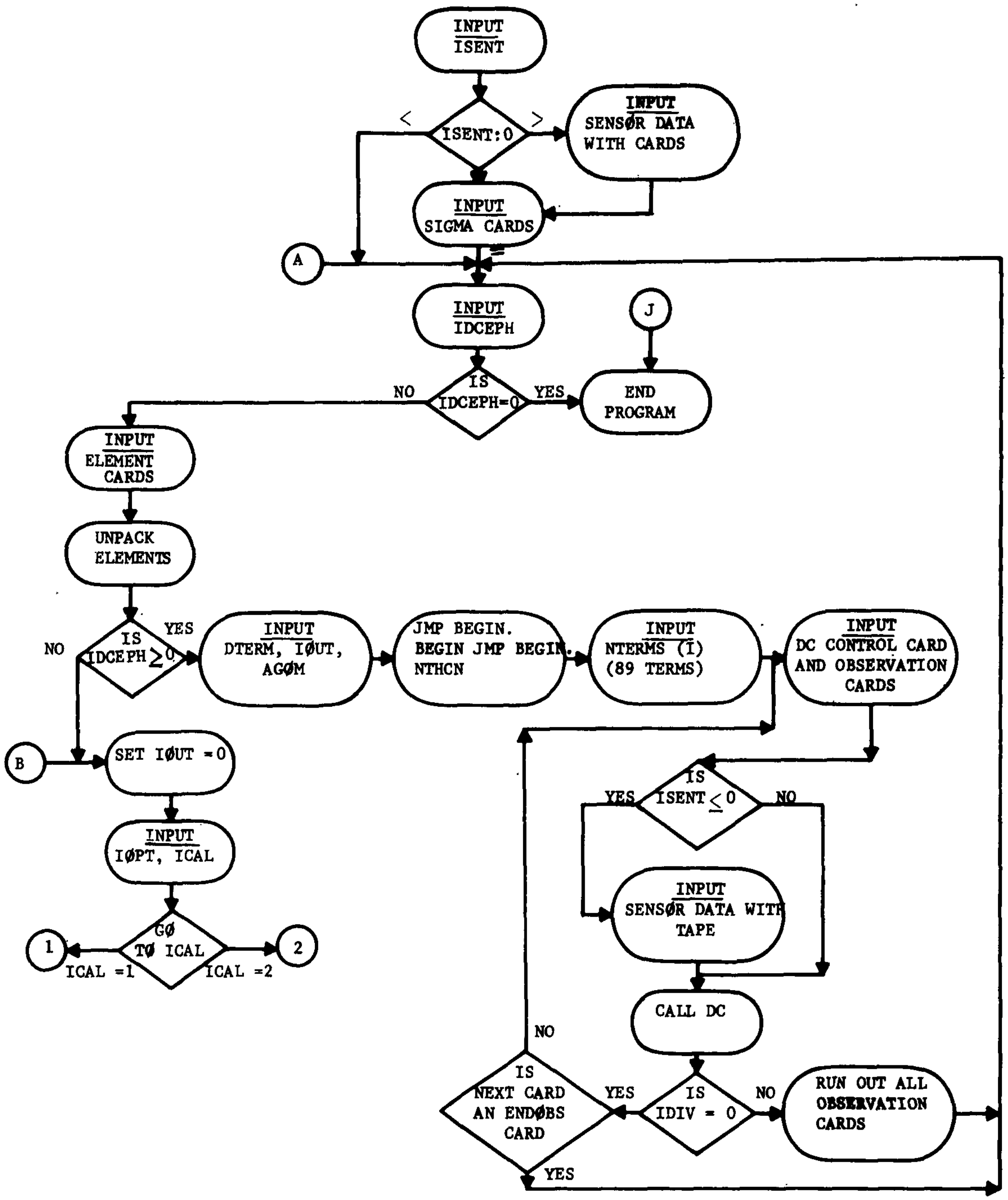
Standard SPADATS Subroutines are used at many places in the program. The flow charts for these are not shown here but a list of their names and reference page numbers (SPADATS manual*) is shown below.

Subroutine	Page	Subroutine	Page
ARCTAN	4-3	PANT	4-29
GLØP	4-29	SEPSUB	4-8
INITEL	4-59	SNSGET	4-65
KLØK	4-9	TLC	4-15
ØBSGET	4-23	XSRCH	4-69
ØBSLØD	4-27		

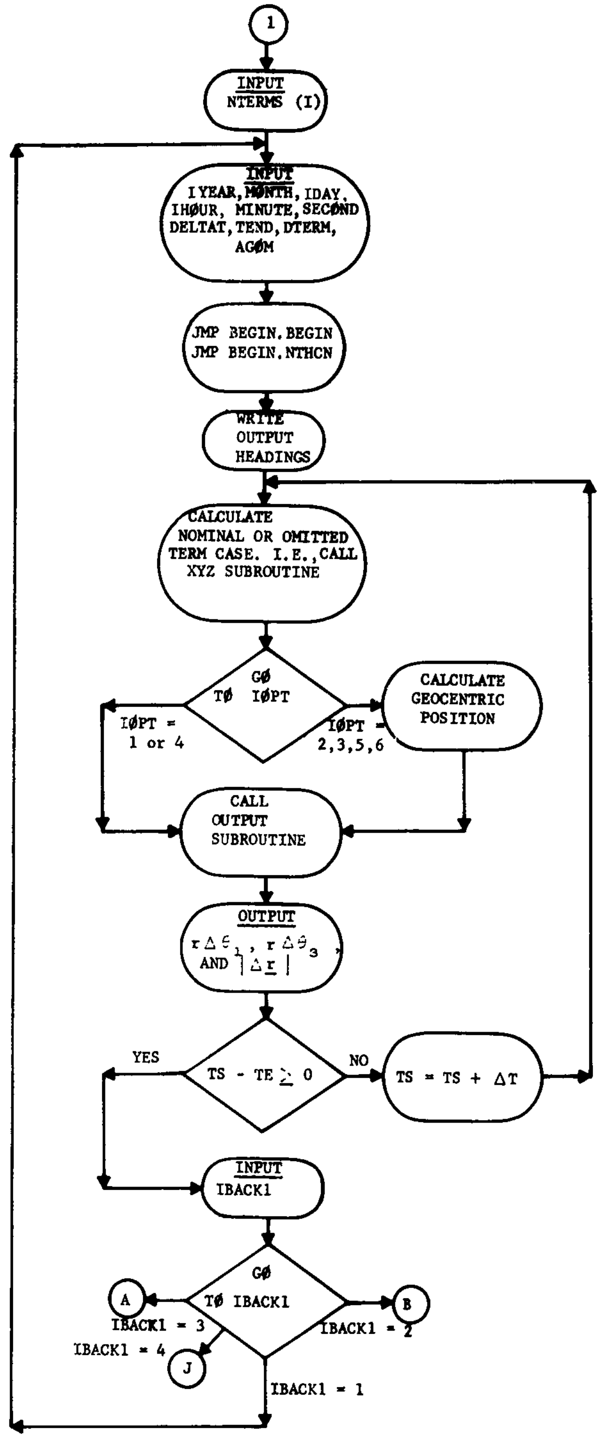
TABLE IV SPADATS SUBROUTINES USED IN PROGRAM

*Aeronutronic Publication U-1691, Revised 1 Obtober 1962

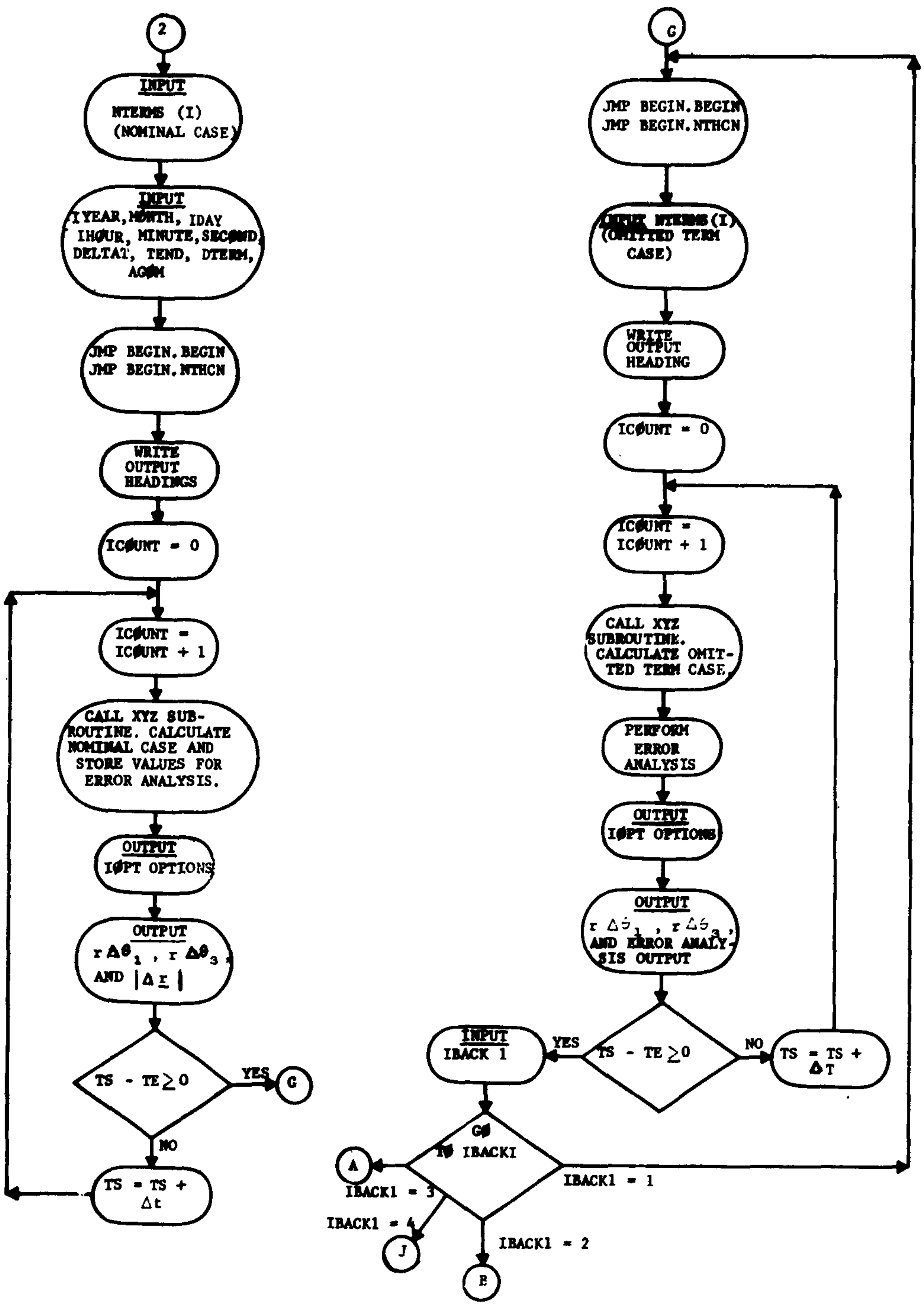
FLOW CHART OF MAIN PROGRAM



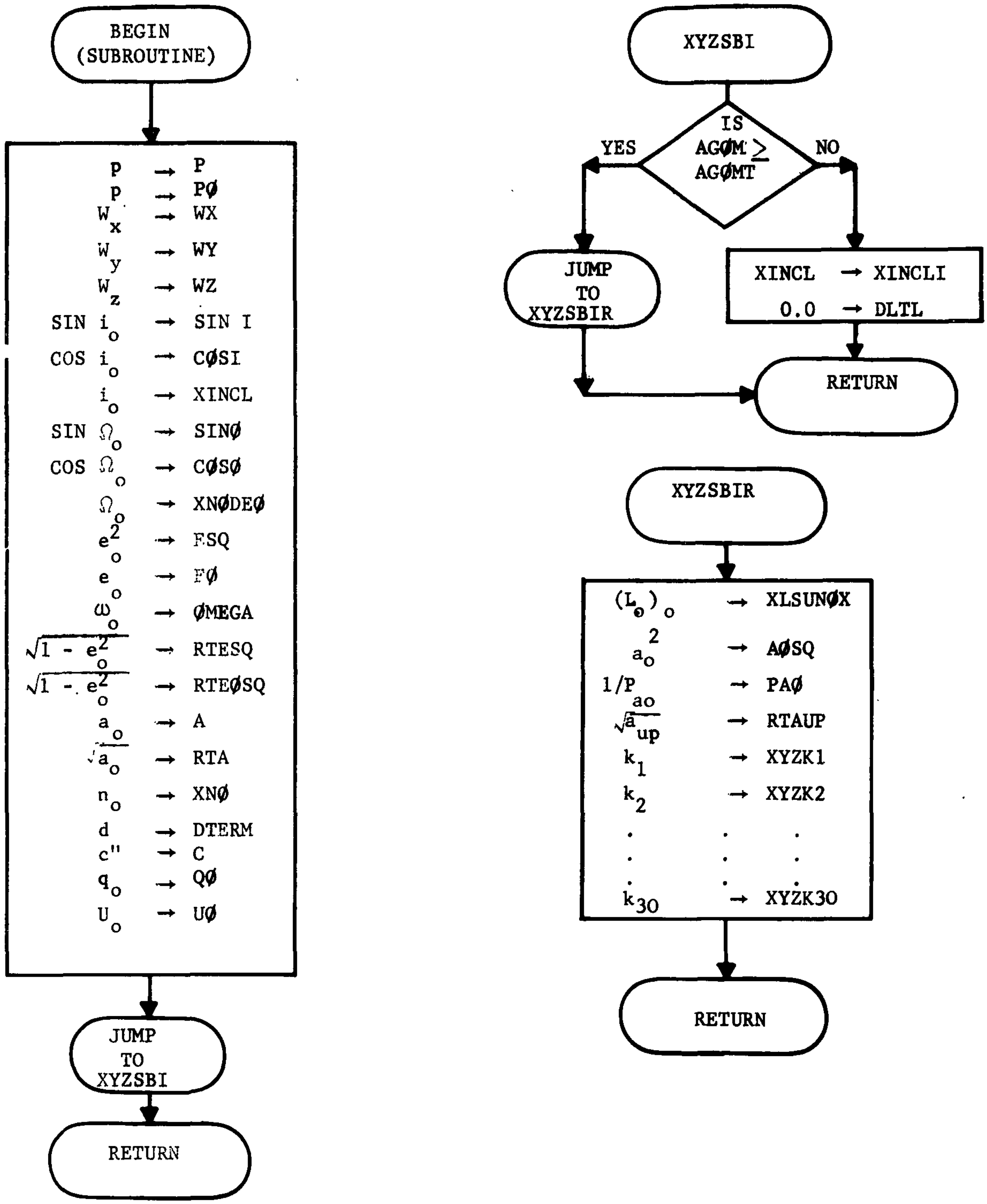
FLOW CHART OF MAIN PROGRAM (Continued)



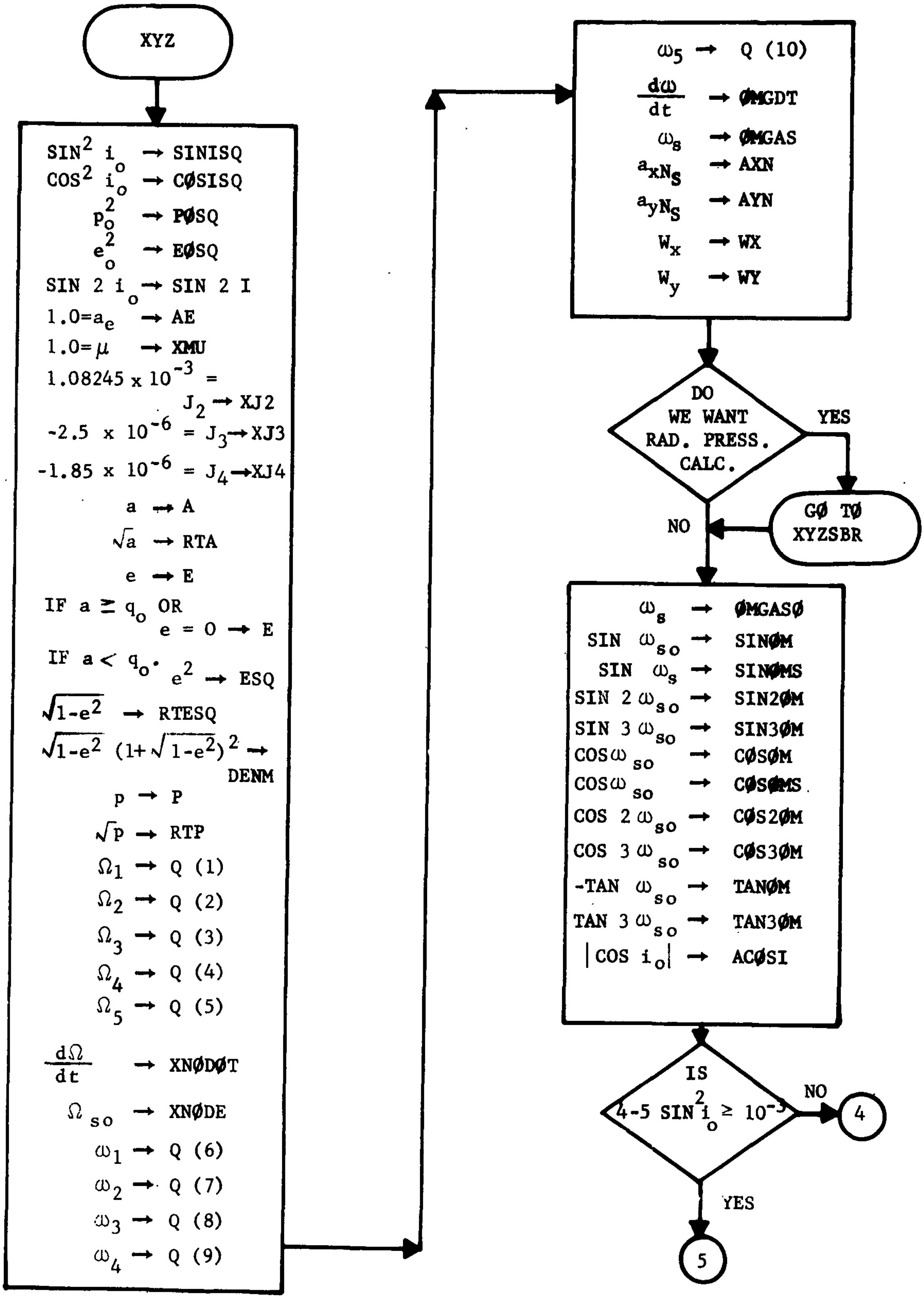
FLOW CHART OF MAIN PROGRAM (continued)



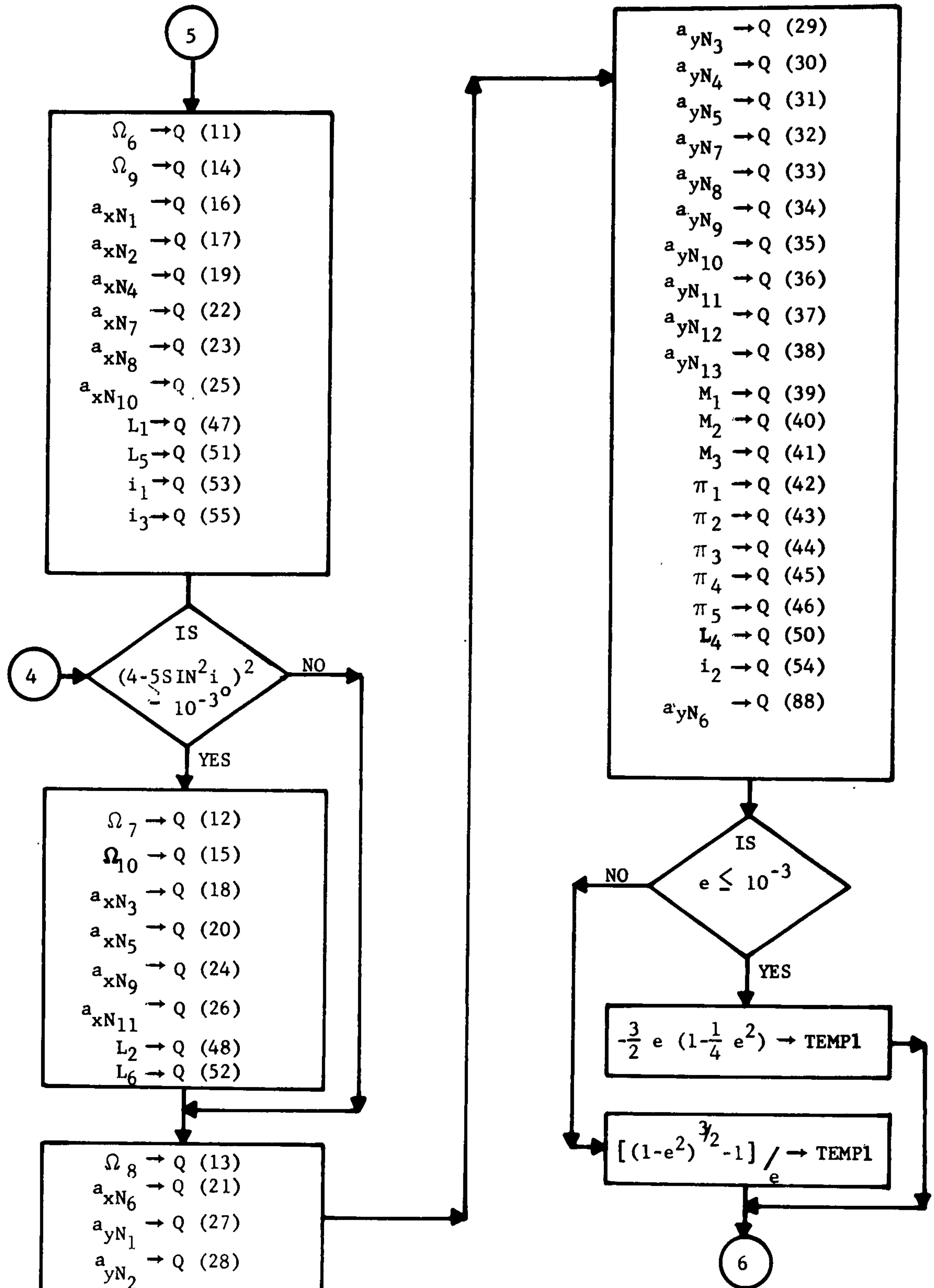
FLOW CHART OF EPHEMERIS INITIALIZING SUBROUTINE



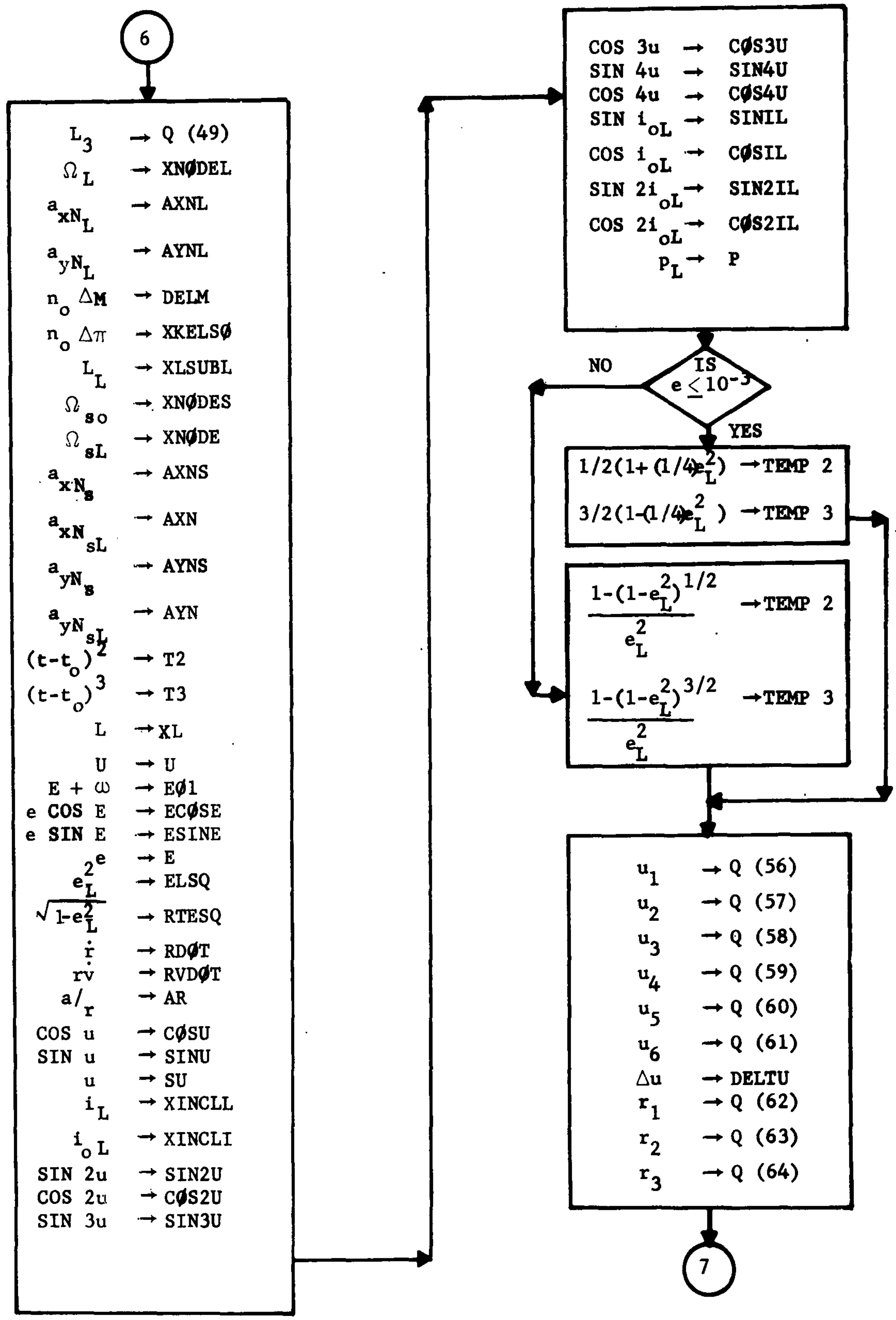
FLOW CHART OF EPHEMERIS SUBROUTINE

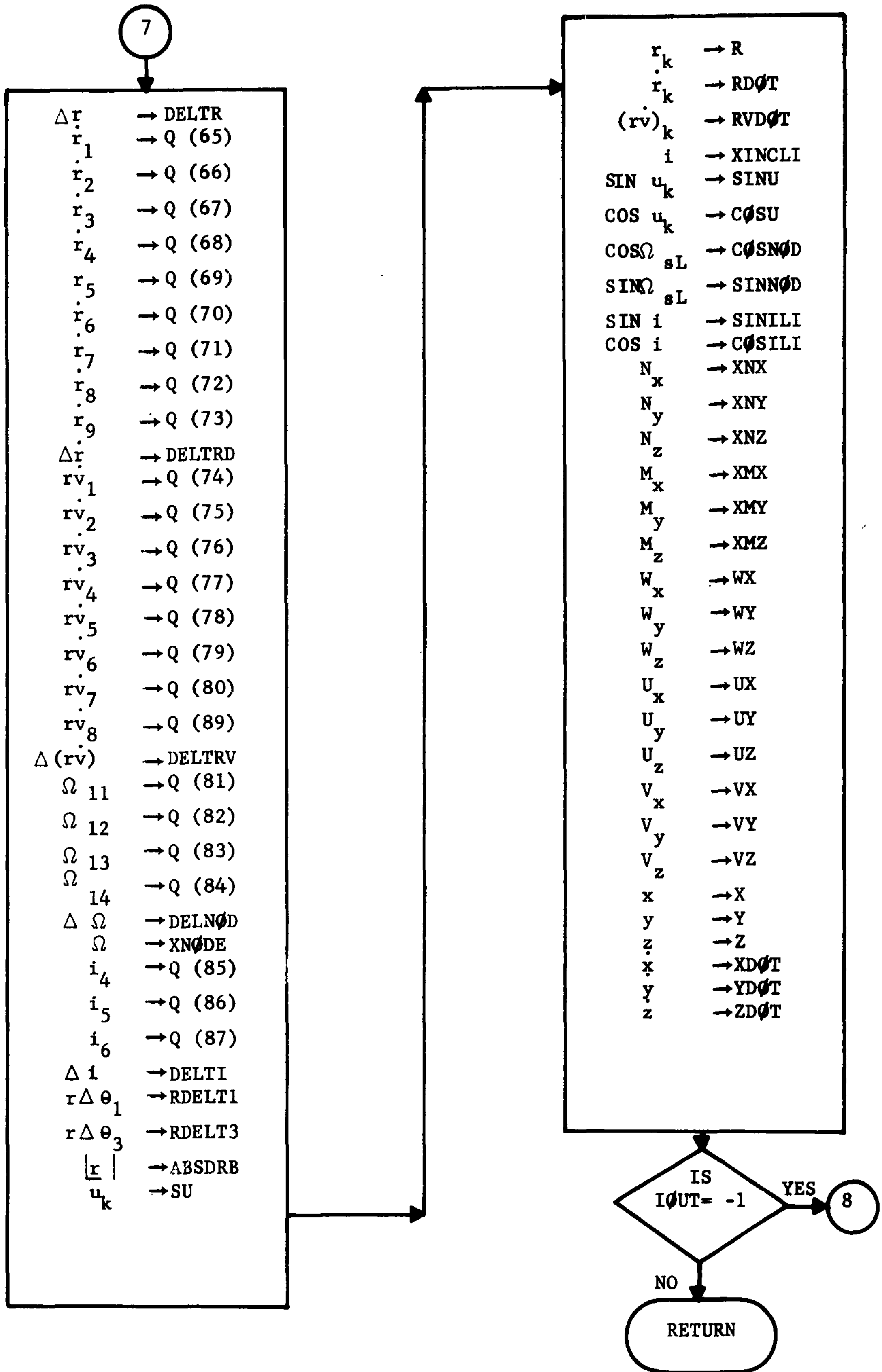


FLOW CHART OF EPHEMERIS SUBROUTINE (CONTINUED)



FLOW CHART OF EPHEMERIS SUBROUTINE (CONTINUED)




7
Δr → DELTR
$\dot{r}_1$ → Q (65)
$\dot{r}_2$ → Q (66)
$\dot{r}_3$ → Q (67)
$\dot{r}_4$ → Q (68)
r_5 → Q (69)
$\dot{r}_6$ → Q (70)
$\dot{r}_7$ → Q (71)
$\dot{r}_8$ → Q (72)
$\dot{r}_9$ → Q (73)
$\Delta\dot{r}$ → DELTRD
$r\dot{v}_1$ → Q (74)
$r\dot{v}_2$ → Q (75)
$r\dot{v}_3$ → Q (76)
$r\dot{v}_4$ → Q (77)
$r\dot{v}_5$ → Q (78)
$r\dot{v}_6$ → Q (79)
$r\dot{v}_7$ → Q (80)
$r\dot{v}_8$ → Q (89)
$\Delta(r\dot{v})$ → DELTRV
Ω_{11} → Q (81)
Ω_{12} → Q (82)
Ω_{13} → Q (83)
Ω_{14} → Q (84)
$\Delta\ \Omega$ → DELNØD
Ω → XNØDE
i_4 → Q (85)
i_5 → Q (86)
i_6 → Q (87)
$\Delta\ i$ → DELTI
$r\Delta\theta_1$ → RDELT1
$r\Delta\theta_3$ → RDELT3
$|\underline{r}|$ → ABSDRB
u_k → SU
r_k → R
$\dot{r}_k$ → RDØT
$(r\dot{v})_k$ → RVDØT
i → XINCLI
SIN u_k → SINU
COS u_k → CØSU
COSΩ_{sL} → CØSNØD
SINΩ_{sL} → SINNØD
SIN i → SINILI
COS i → CØSILI
N_x → XNX
N_y → XNY
N_z → XNZ
M_x → XMX
M_y → XMY
M_z → XMZ
W_x → WX
W_y → WY
W_z → WZ
U_x → UX
U_y → UY
U_z → UZ
V_x → VX
V_y → VY
V_z → VZ
x → X
y → Y
z → Z
$\dot{x}$ → XDØT
$\dot{y}$ → YDØT
$\dot{z}$ → ZDØT
IS IØUT= -1
YES
8
NO
RETURN

8

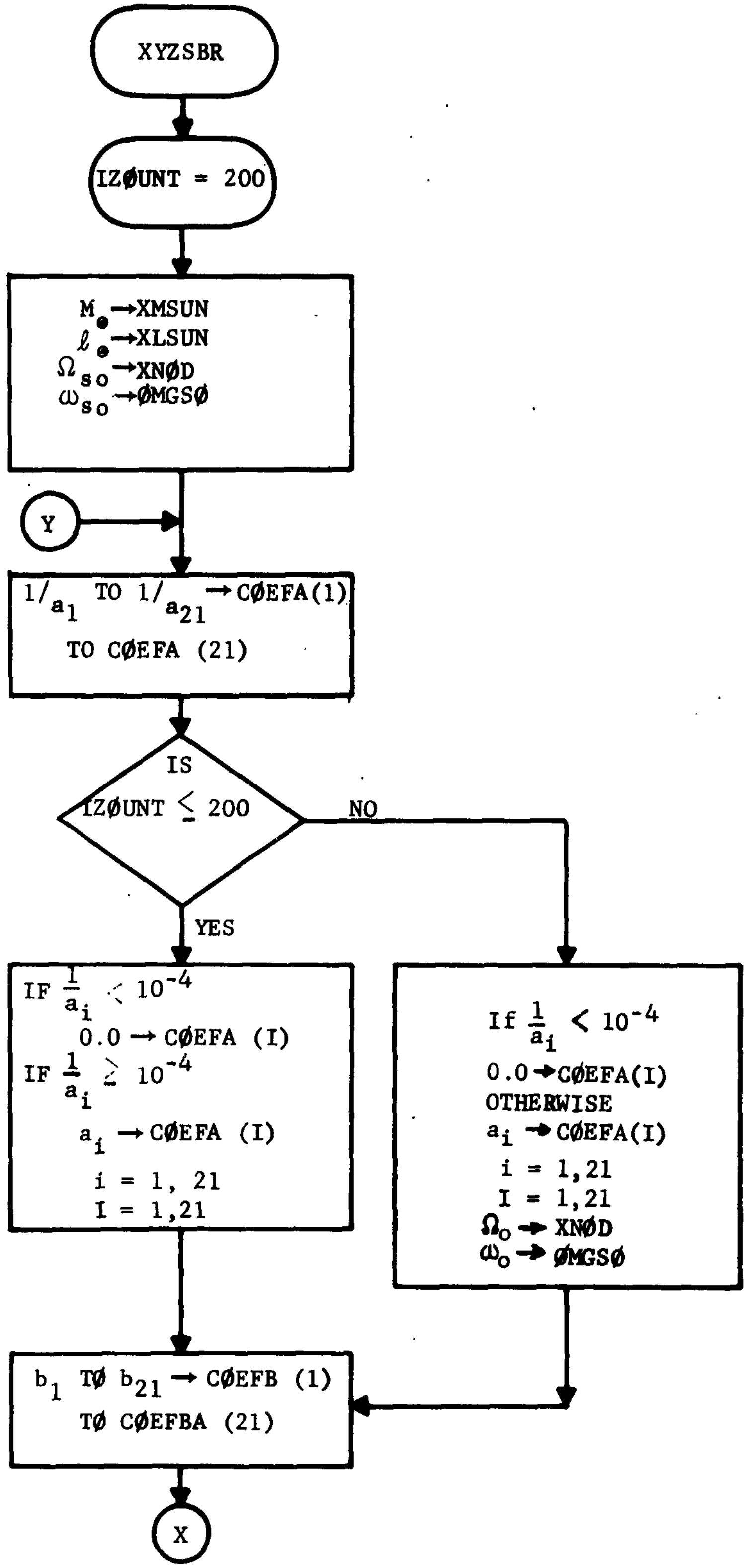
OUTPUT THE FOLLOWING QUANTITIES FOR OFFLINE PRINTING:

SATELLITE NUMBER, SATELLITE NAME, ELEMENT SET NUMBER, T_o, TIME OF EPOCH, THE INDIVIDUAL VALUES OF THE 89 GENERAL PERTURBATION TERMS,

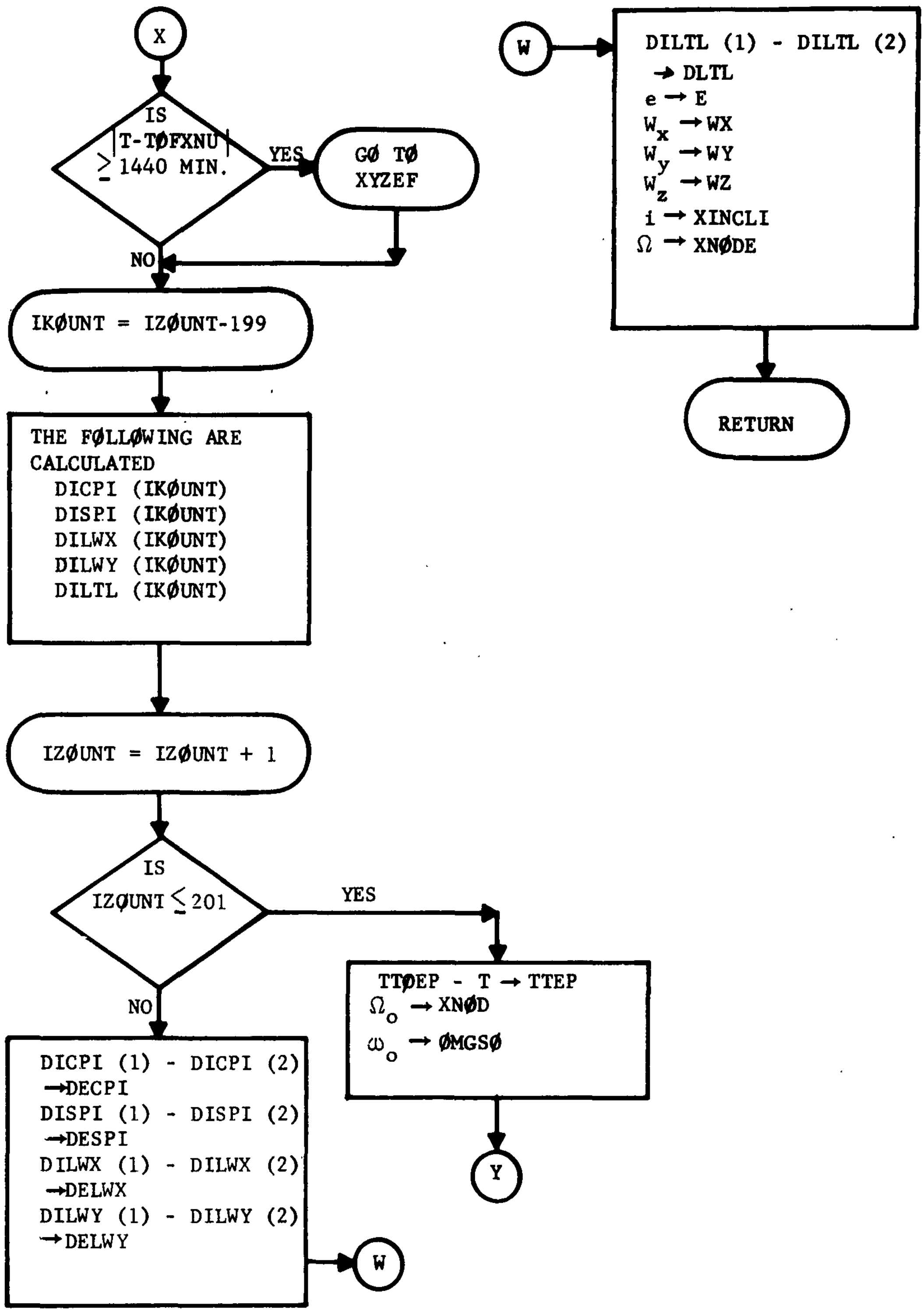
a_{xN_s}, a_{xN_L}, $a_{xN_{sL}}$, a_{yN_s},
$a_{y_{NL}}$, $a_{y_{NsL}}$, Ω_s, Ω_L, Ω_{sL},
$\Delta\Omega$, ω_{so}, L_L, n_o $\Delta\pi$,
$n_o \Delta M$, i_L, i, Δi, e_L,
x, y, z, $\dot{x}$, $\dot{y}$, $\dot{z}$, u_k,
Δu, r_k, Δr, $\Delta \dot{r}$,
$\Delta(r\dot{v})$, $\dot{r}_k$, $r\Delta\theta_1$,
$r \Delta \theta_3$, $|\underline{r}|$

RETURN

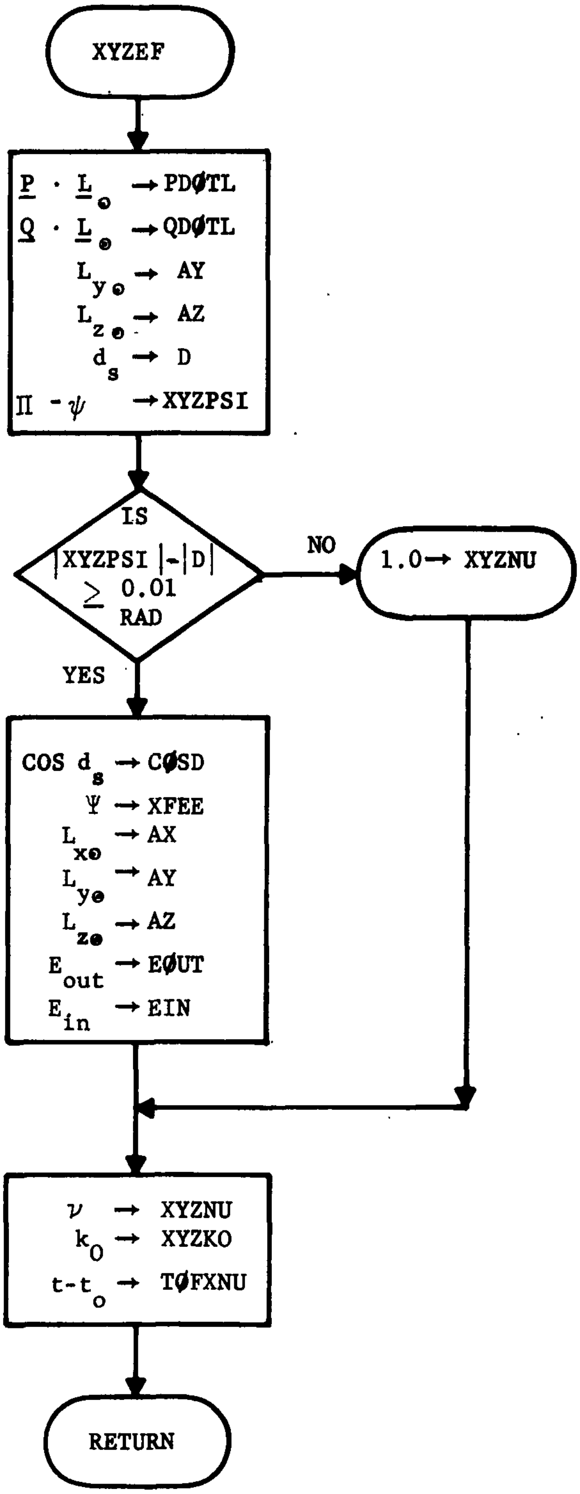
FLOW CHART OF RADIATION PRESSURE SECTION OF EPHEMERIS SUBROUTINE



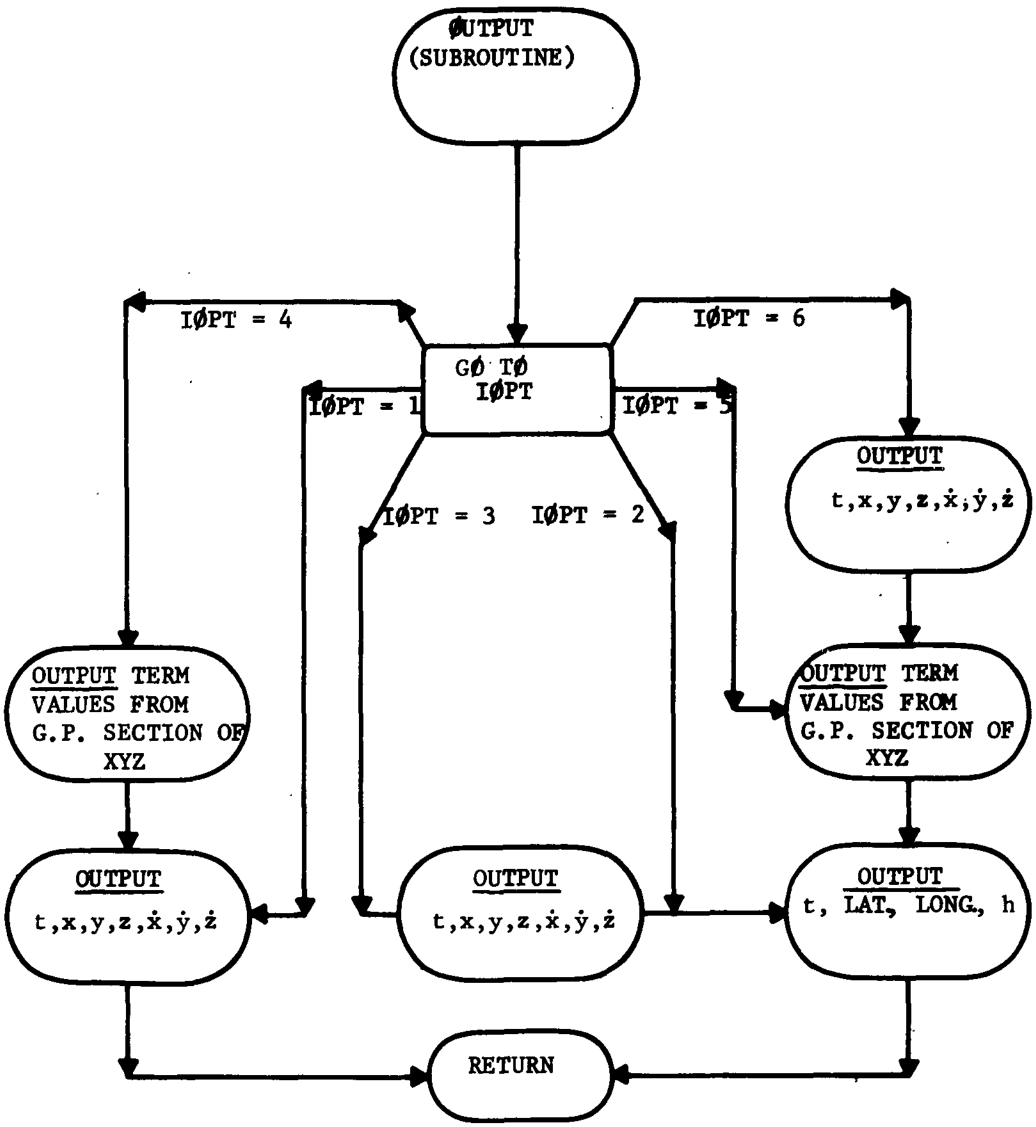
FLOW CHART OF RADIATION PRESSURE SECTION
OF EPHEMERIS SUBROUTINE (CONTINUED)



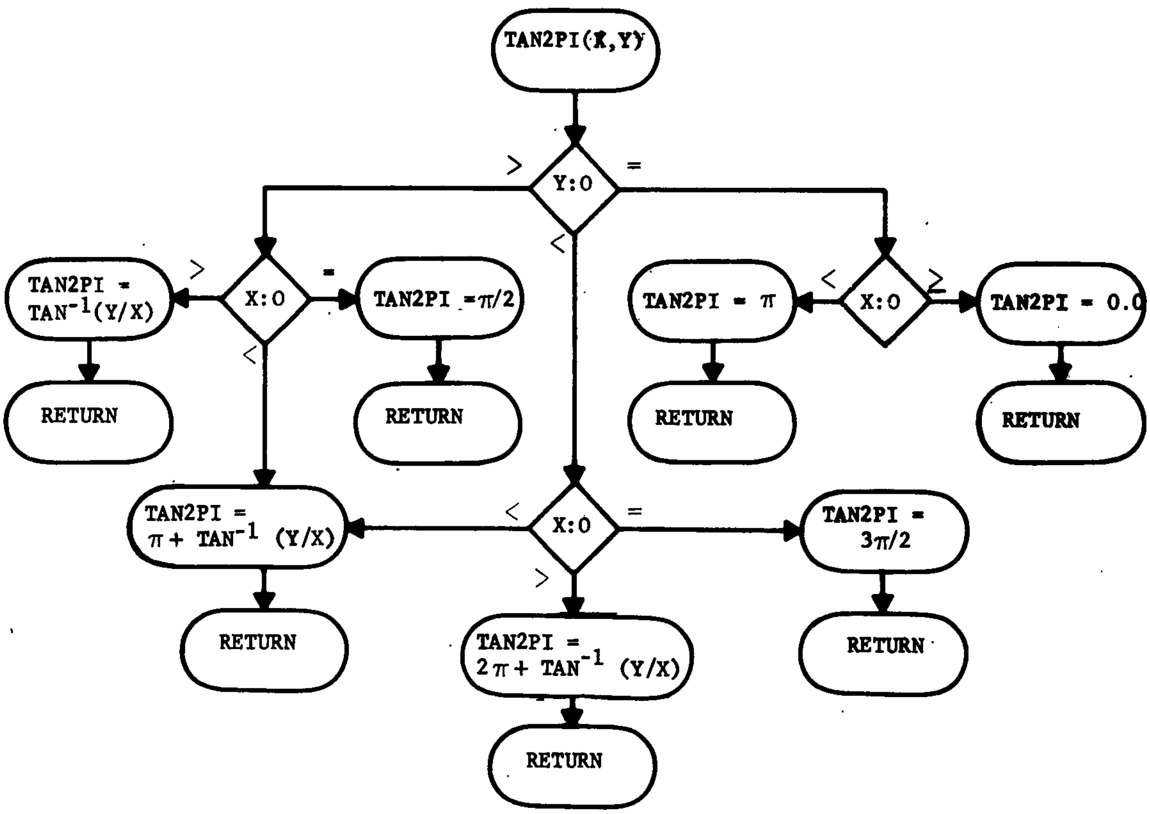
FLOW CHART OF RADIATION PRESSURE SECTION OF EPHEMERIS SUBROUTINE (CONTINUED)



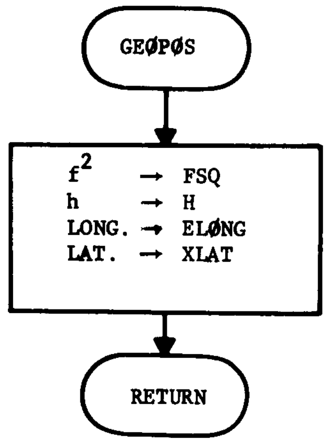
FLOW CHART OF OUTPUT SUBROUTINE



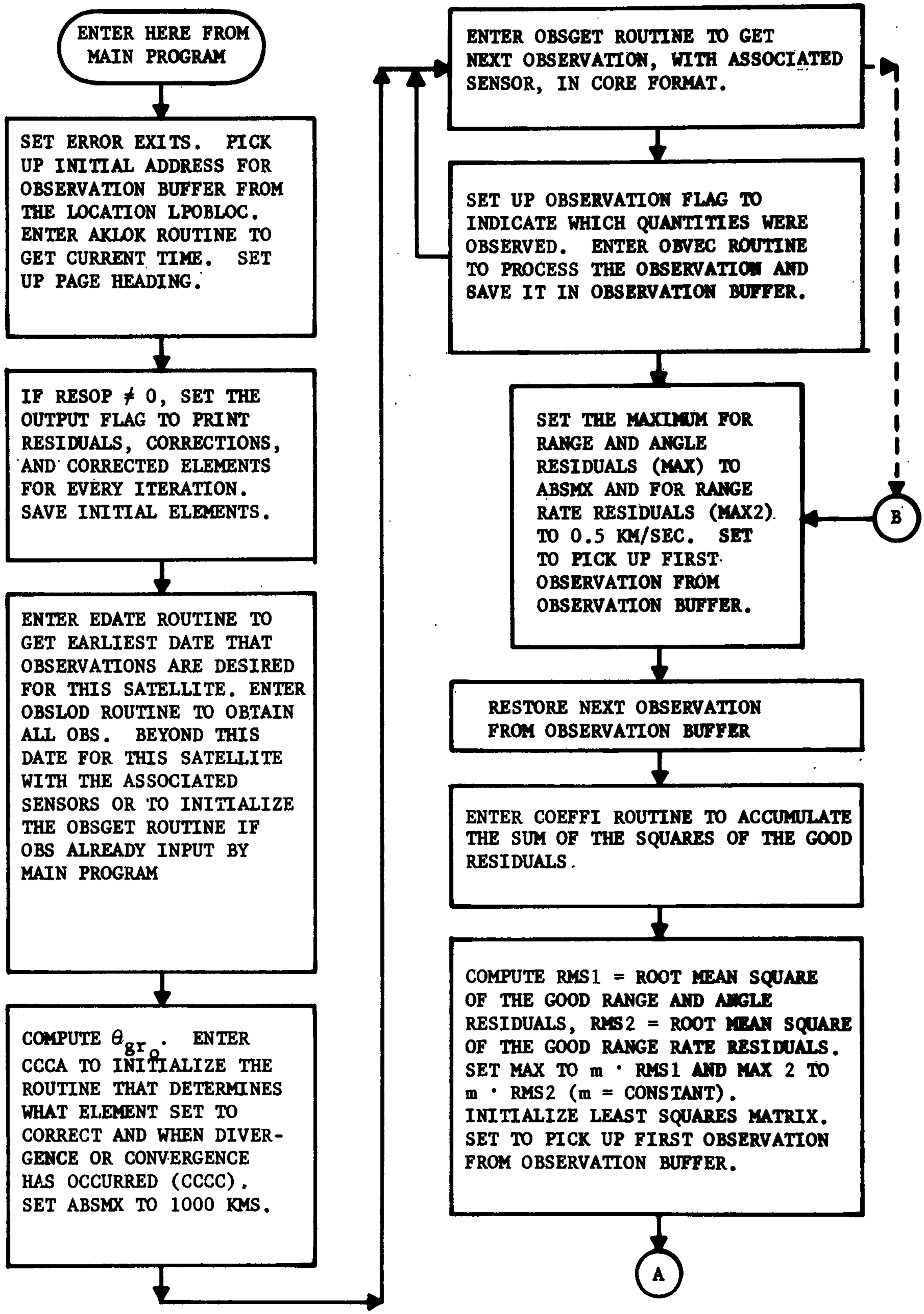
FLOW CHART OF ARCTANGENT SUBROUTINE



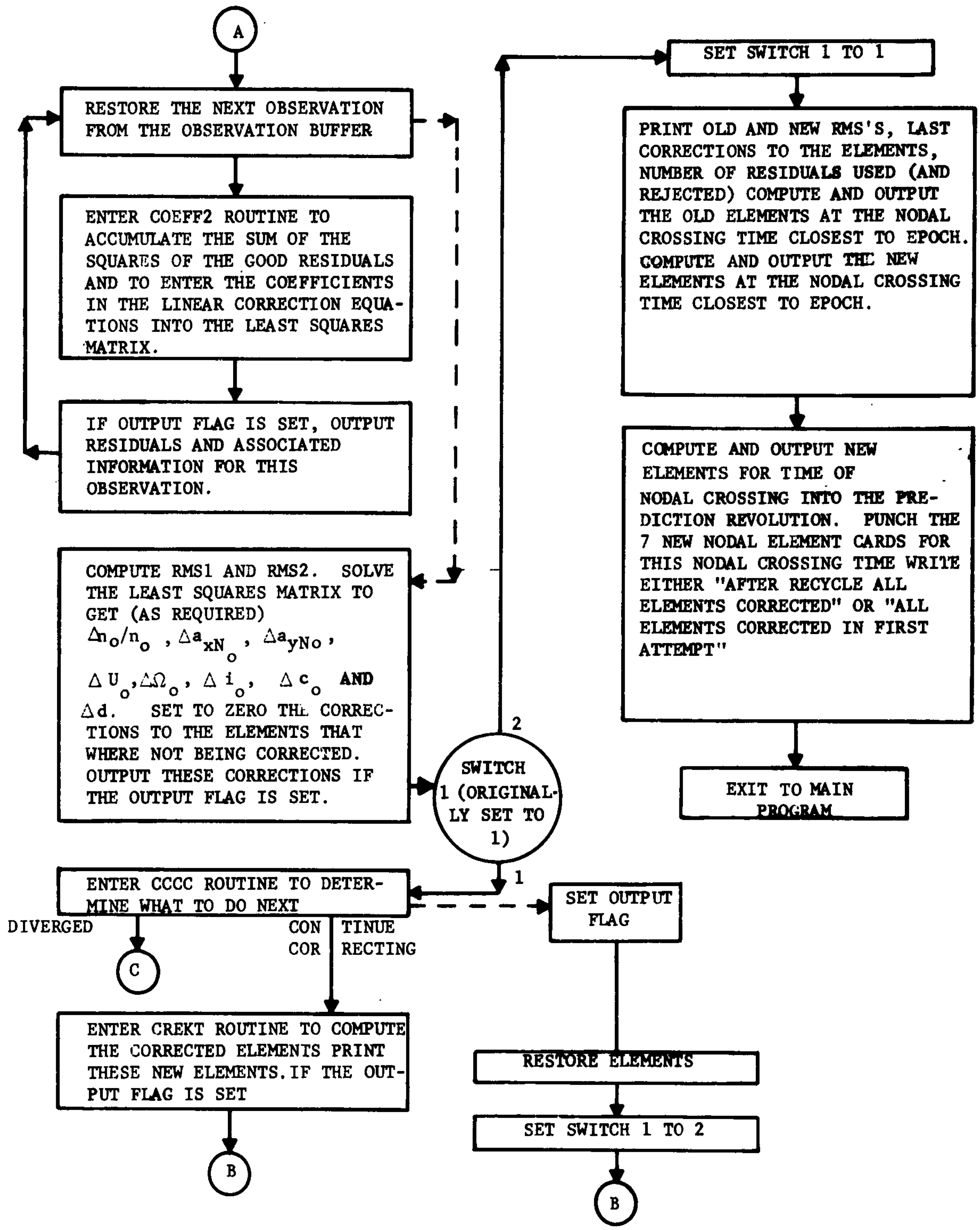
FLOW CHART OF GEOCENTRIC POSITION SUBROUTINE



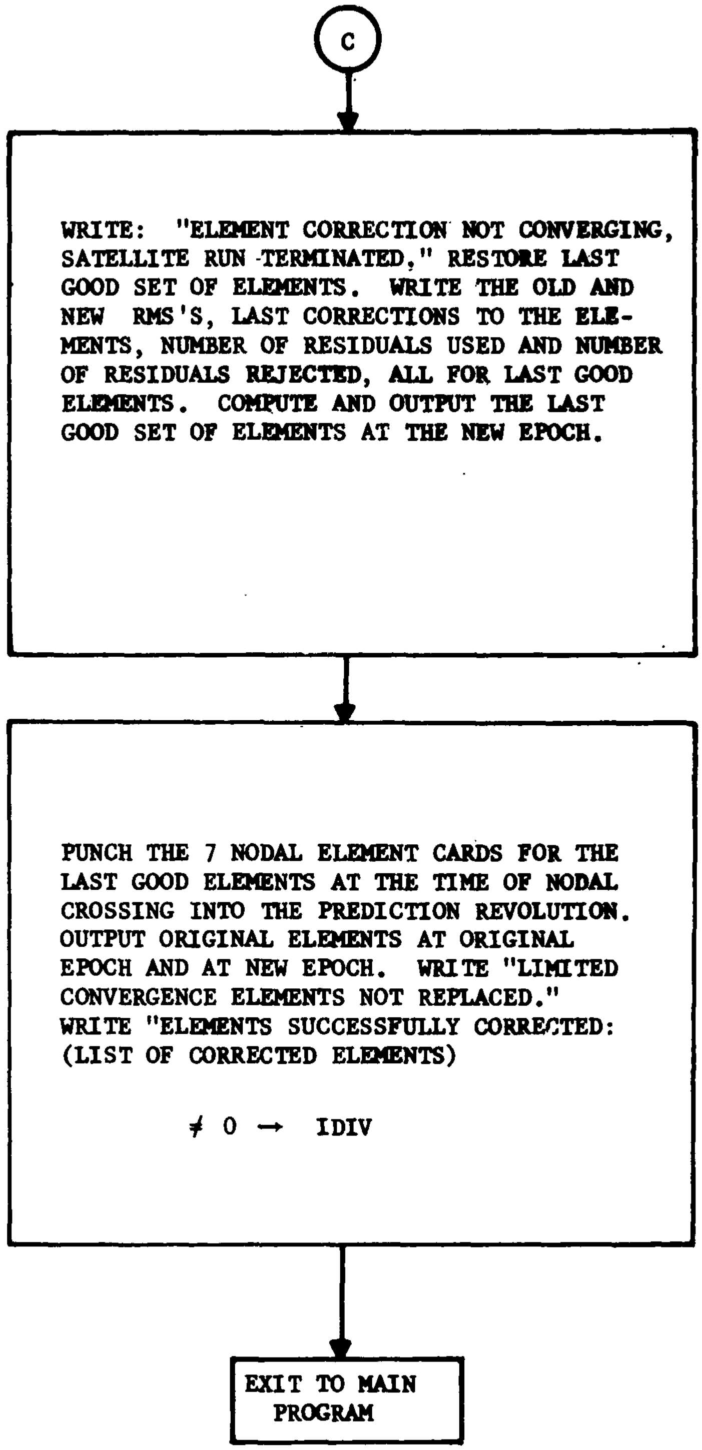
FLOW CHART OF DIFFERENTIAL CORRECTION (D.C.) SUBROUTINE



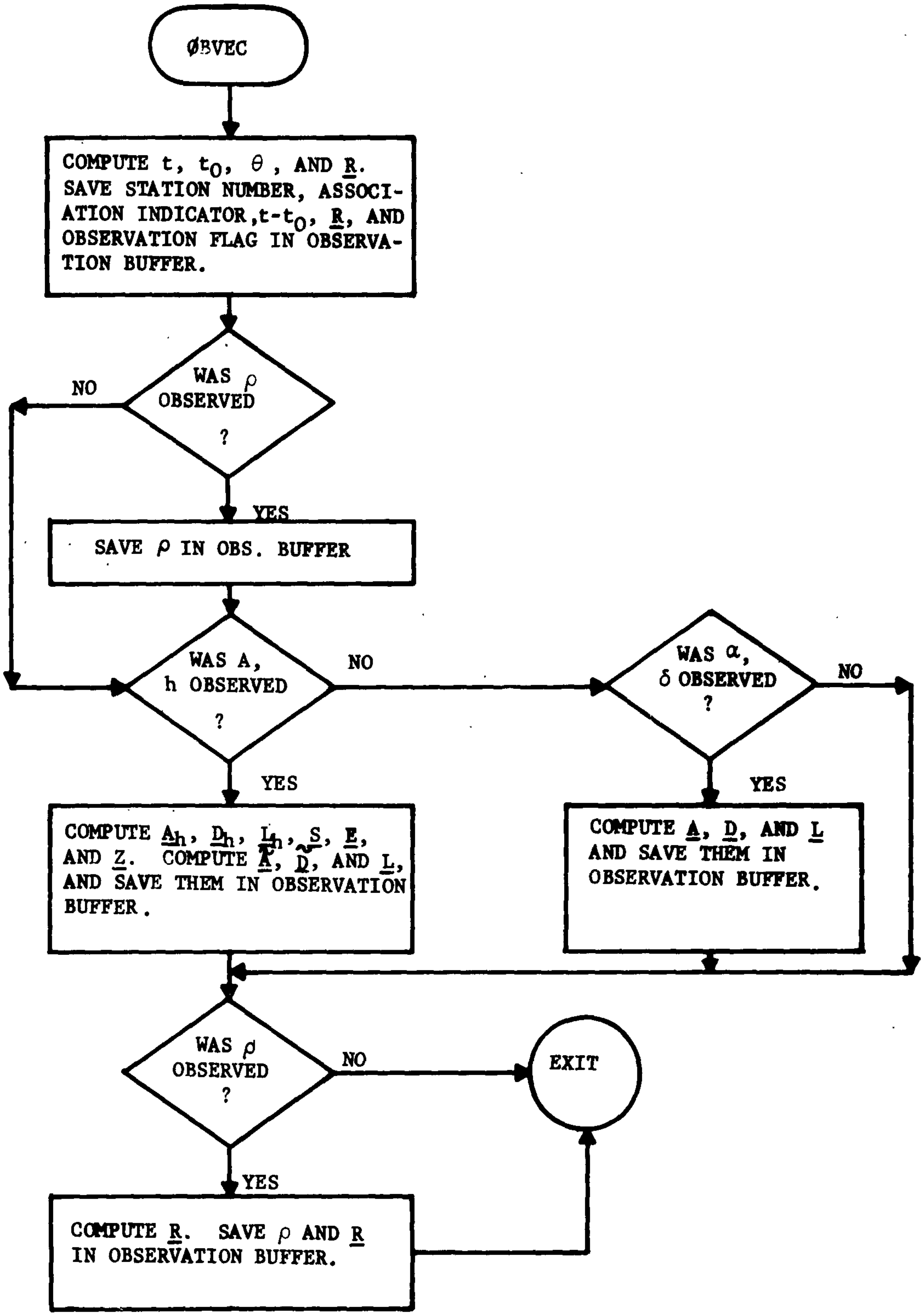
FLOW CHART OF D.C. SUBROUTINE (CONTINUED)



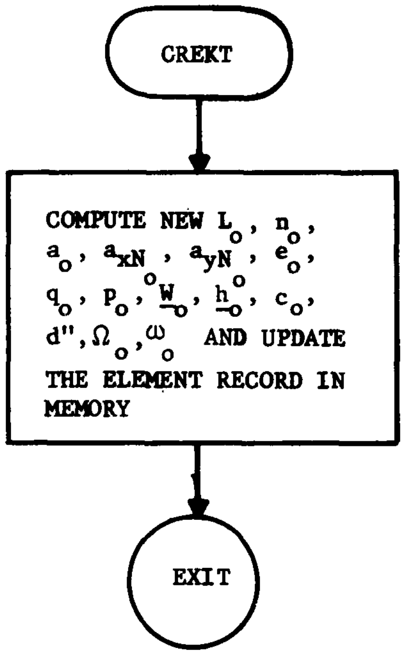
FLOW CHART OF D.C. SUBROUTINE (CONTINUED)



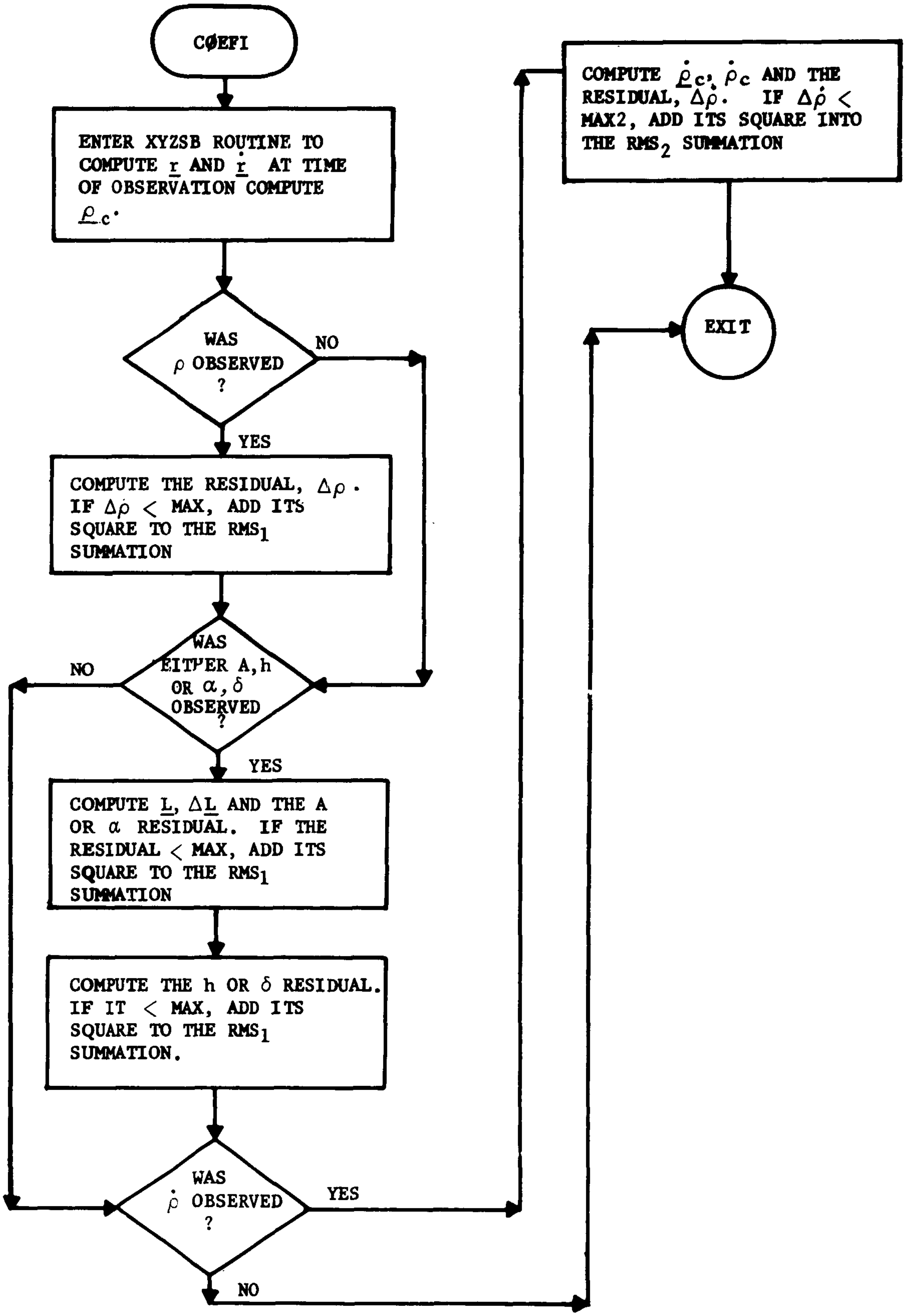
FLOW CHART OF D.C. SUBROUTINE (CONTINUED)



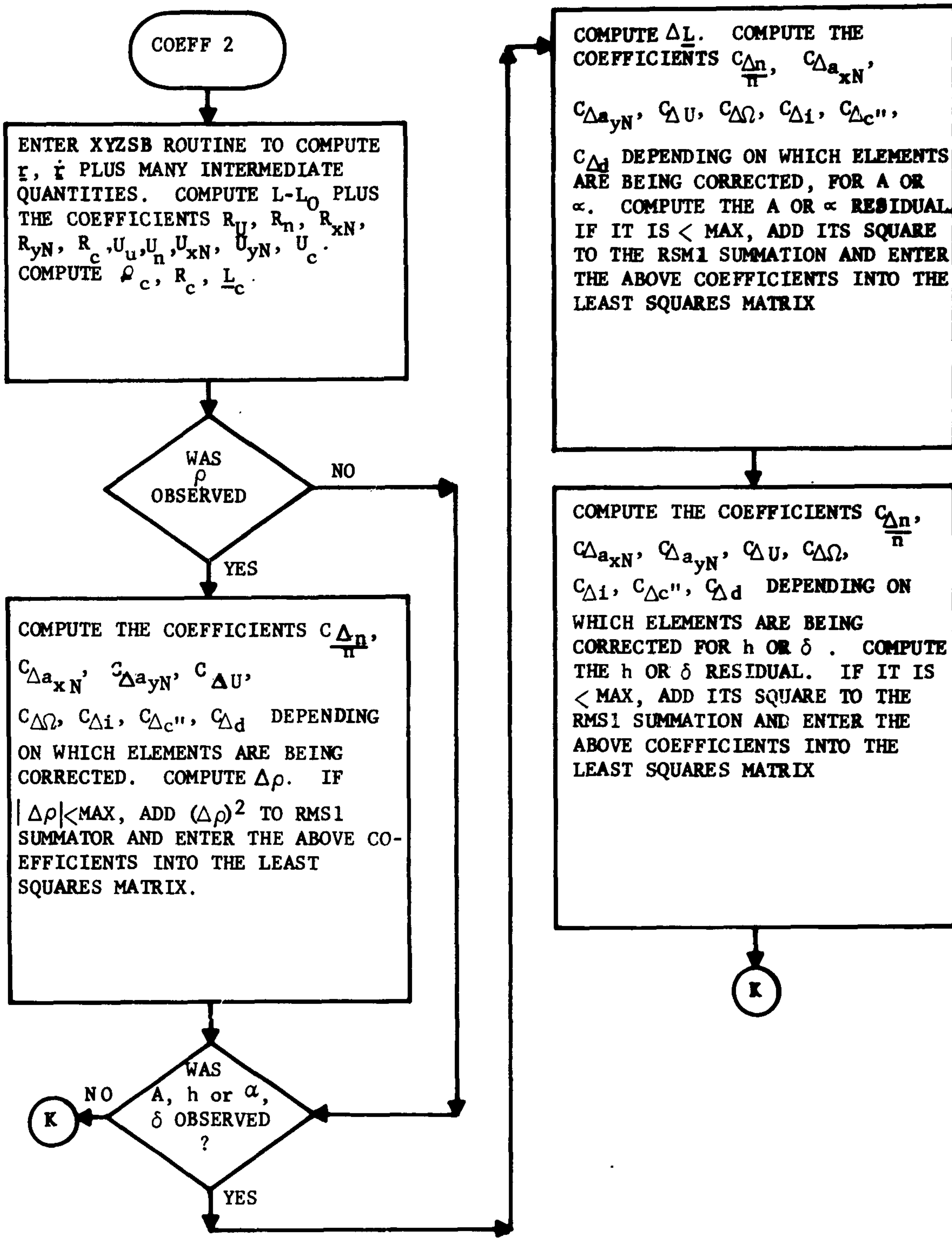
FLOW CHART OF D.C. SUBROUTINE (CONTINUED)



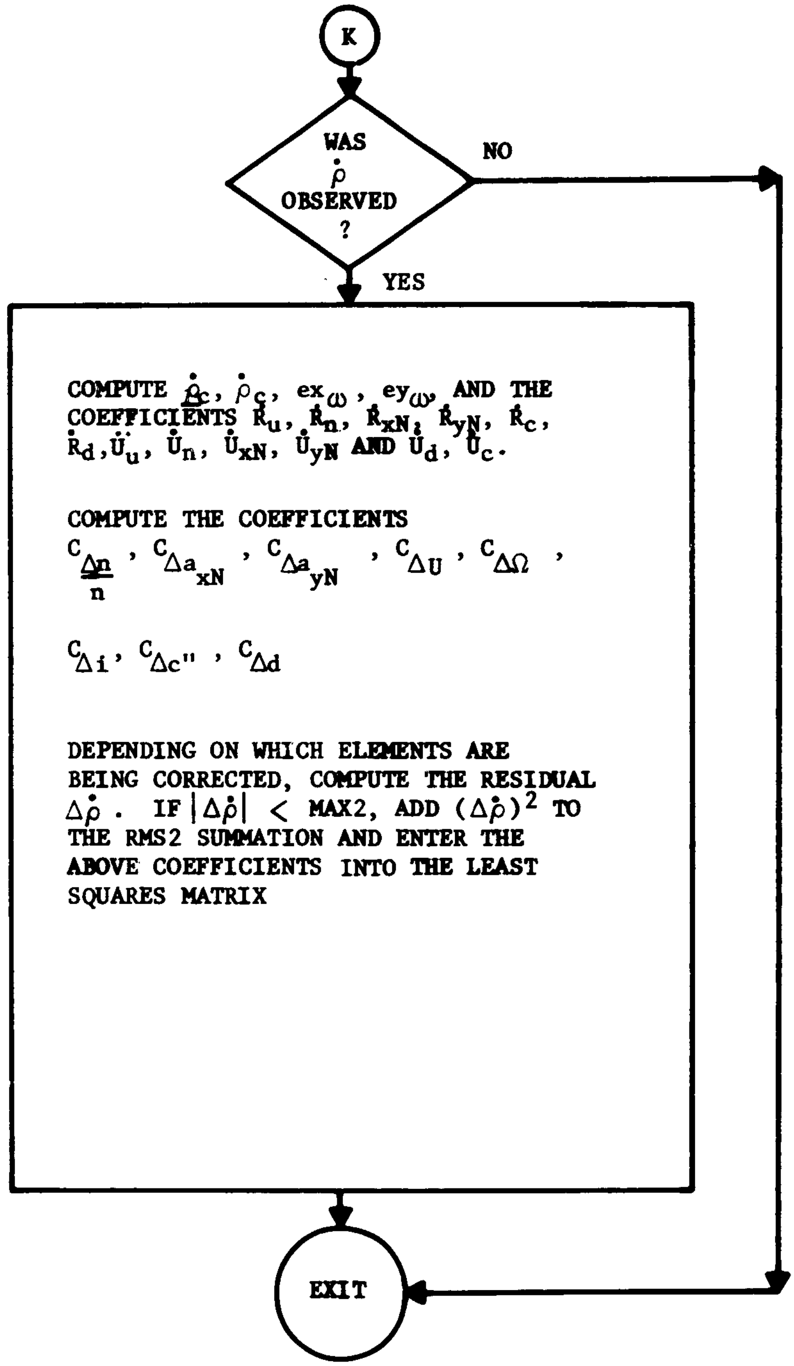
FLOW CHART OF D.C. SUBROUTINE (CONTINUED)



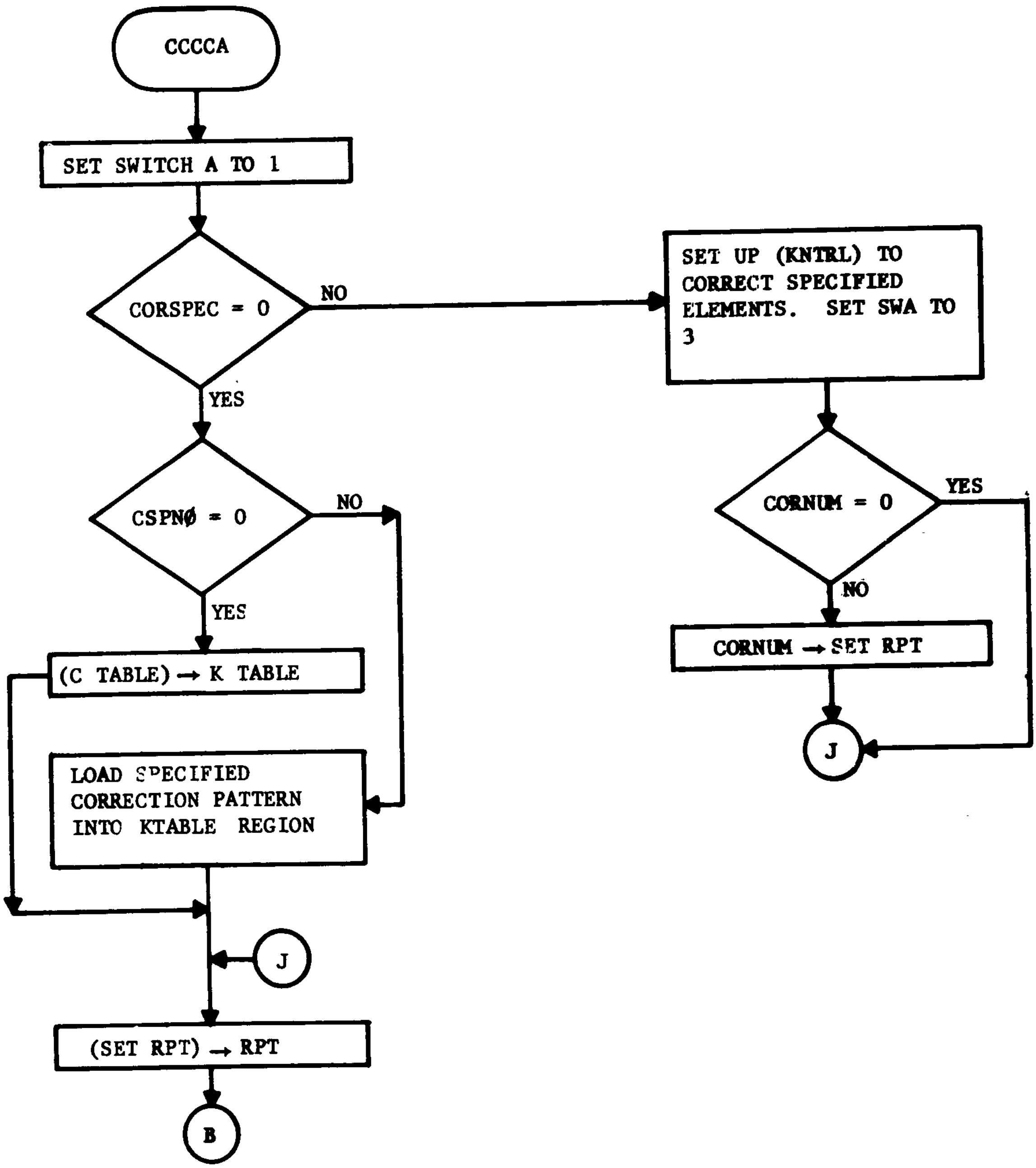
FLOW CHART OF D.C. SUBROUTINE (CONTINUED)



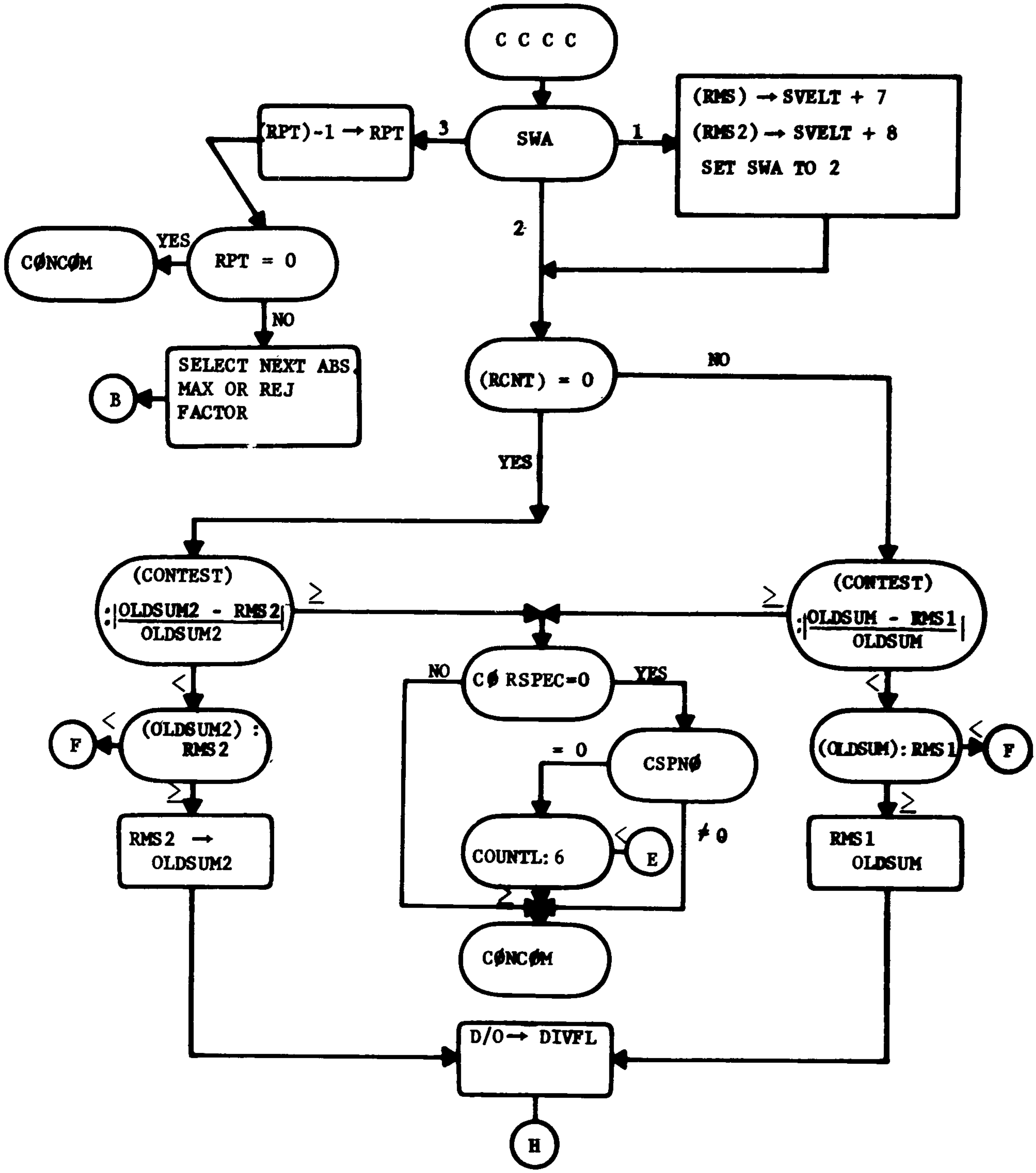
FLOW CHART OF D.C. SUBROUTINE (CONTINUED)



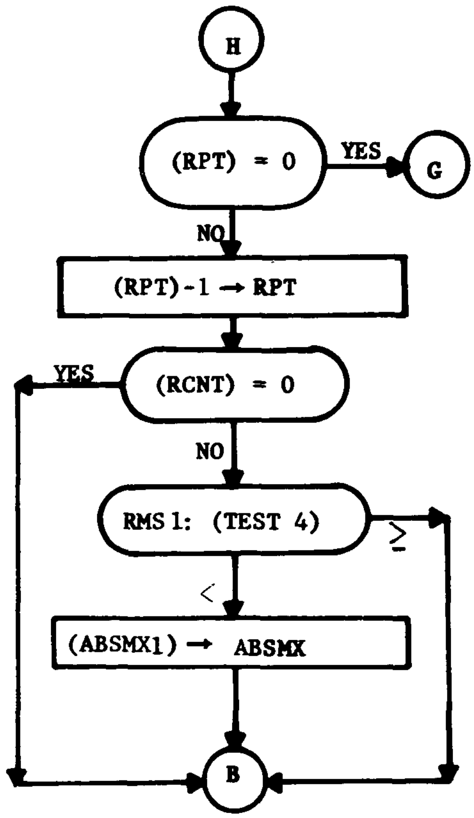
FLOW CHART OF D.C. SUBROUTINE (CONTINUED)



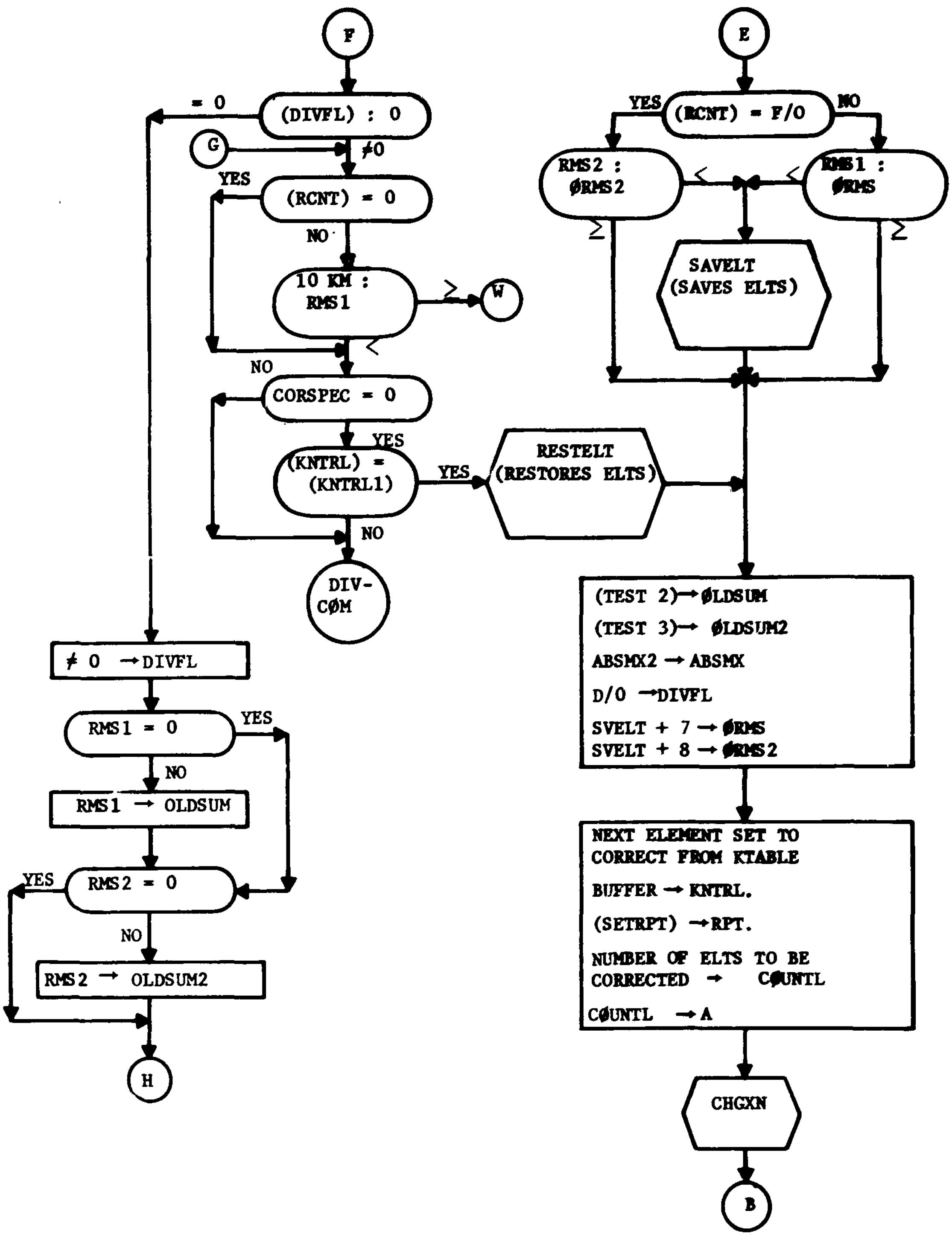
FLOW CHART OF D.C. SUBROUTINE (CONTINUED)



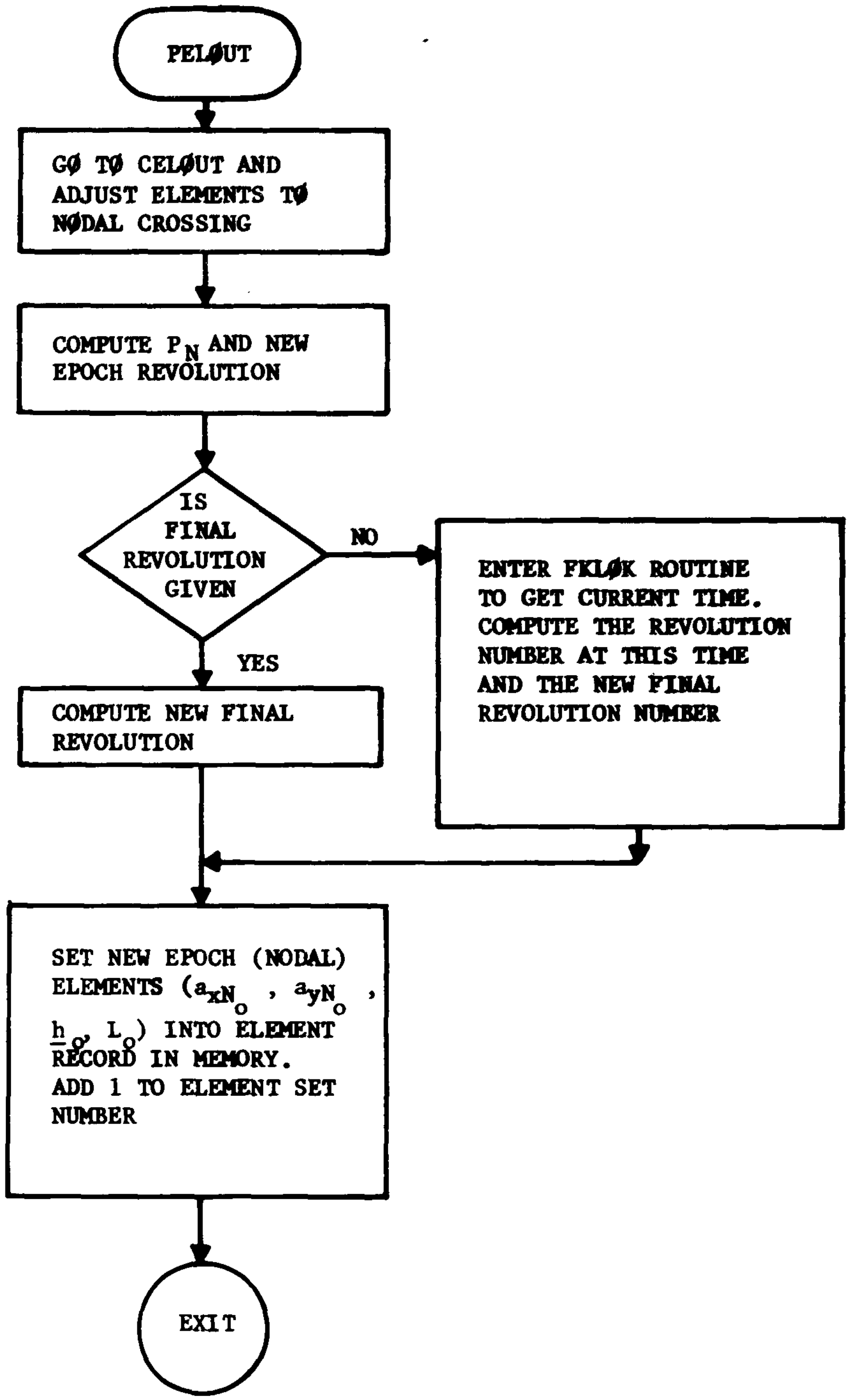
FLOW CHART OF D.C. SUBROUTINE (CONTINUED)



FLOW CHART OF D.C. SUBROUTINE (CONTINUED)



FLOW CHART OF D.C. SUBROUTINE (CONTINUED)



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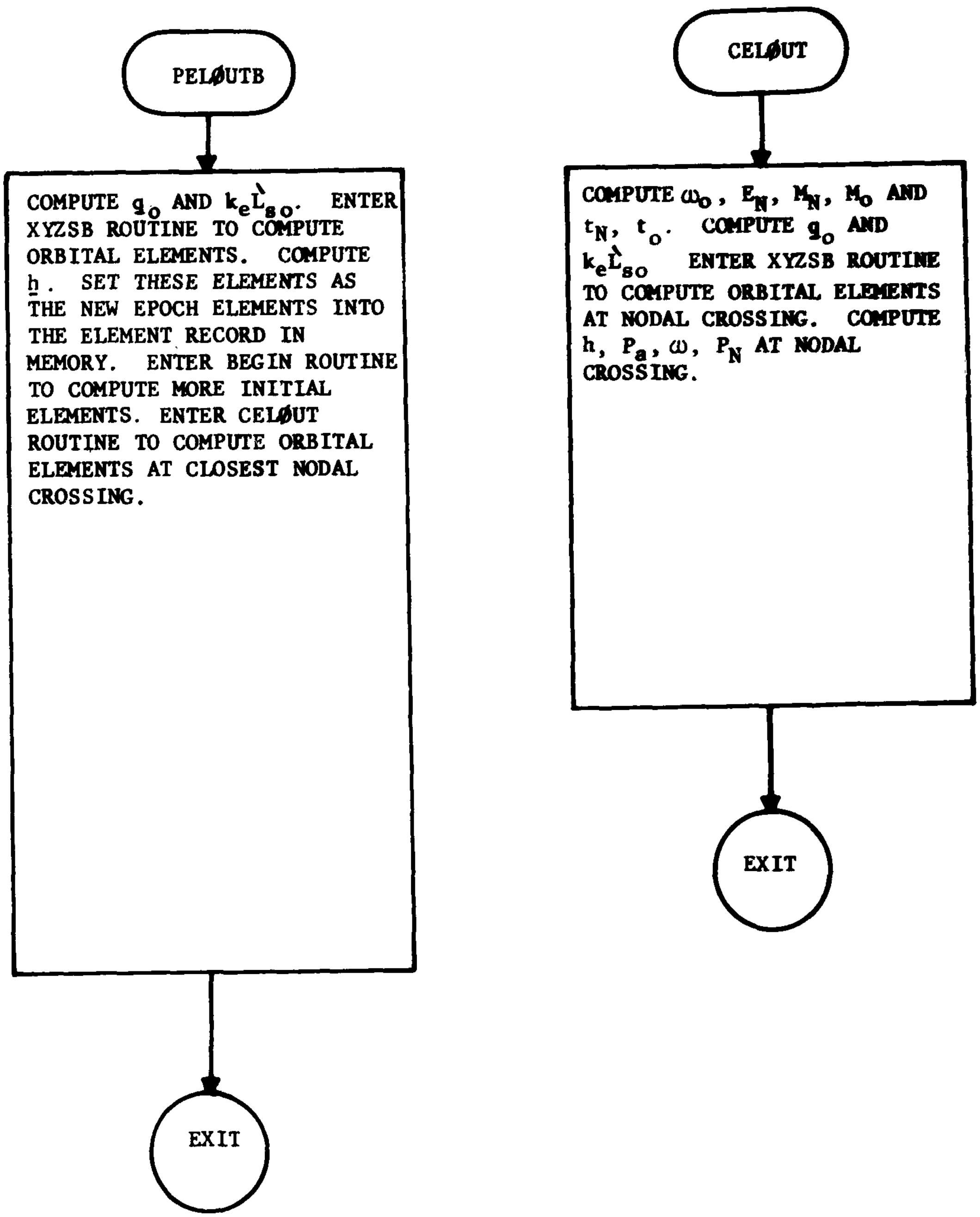

3.6 Program Symbol Definitions

This section shows many of the symbolic locations used in the program and the quantities which they contain. The units of the 89 General Perturbations terms, Q(1) to Q(89), are also shown.

SYMBOL	CONTENTS		SYMBOL	CONTENTS
A	a	(earth radii (E.R.)	CØSI	$\cos i_o$
ABSDRB	$\lvert \Delta \underline{r} \rvert$	(km)	CØSII	$\cos i$
AE	a_e	(E.R.))	COSIL	$\cos i_{oL}$
AGØM	$\frac{A}{m} \gamma$	$\left(\frac{cm^2}{gm} \right)$	CØSILI	$\cos i$
AØ	a_o	(E.R.)	COSILS	$\cos^2 i_{oL}$
AXN	$a_{xN_{SL}}$		COSI4	$\cos^4 i_o$
AYN	$a_{yN_{SL}}$		COS2IL	$\cos 2i_{oL}$
AXNL	a_{xN_L}		COSNØD	$\cos \Omega$
AYNL	a_{yN_L}		CØSØM	$\cos \omega_{so}$
AXNØ	a_{xN_o}		CØS2ØM	$\cos 2\omega_{so}$
AYNØ	a_{yN_o}		COS3ØM	$\cos 3\,\omega_{so}$
AXNS	a_{xN_S}		CØSU	$\cos u$
AYNS	a_{yN_S}		CØSUN	$\cos u_n$
C	c"	(1/min)	CØS2U	$\cos 2u$
CØ	c_o	(days/rev^2)	CØS3U	$\cos 3u$
CØSEØ	$\cos (E+\omega)$		CØS4U	$\cos 4u$

SYMBOL	CONTENTS	
DELIE*	Δi	(rad.)
DELM	$n_o \Delta M$	
DELNØD	$\Delta\Omega$	(rad.)
DELNØE*	$\Delta\Omega$	"
DELRE*	Δr	(km)
DELTAT	Δt	(min)
DELTI	Δi	(rad.)
DELTR	Δr	(E.R.)
DELTRD	$\Delta \dot{r}$	(E.R./k_e^{-1} min)
DELTRV	$\Delta (r\dot{v})$	(E.R./k_e^{-1} min)
DELTU	Δu	(rad.)
DELUE*	Δu	"
DTERM	d	(1/min^2)
E	e	
ECØSE	e cos E	
ELSQ	e_L^2	
EØ	e_o	
EØ1	$(E+\omega)$	(rad.)
EØSQ	e_o^2	

SYMBOL	CONTENTS	
ESINE	e sin E	
ESQ	e^2	
FLØ8	A	
HXØ	h_{x_o}	(E.R.)$^{\frac{1}{2}}$
HYØ	h_{y_o}	"
HZØ	h_{z_o}	"
NDECAY	n_D	
ØMEGA	ω_o	(rad.)
ØMGAS	ω_s	"
OMGASØ	ω_{so}	"
ØMGDT	$\frac{d\omega}{dt}$	(rad./min.)
P	p	(E.R.)
P	p_L	"
PI	π	
PØ	p_o	(E.R.)
QØ	q_o	"
Q(1)	Ω_1	(rad./min.)
Q(2)	Ω_2	"
Q(3)	Ω_3	"

*These symbols are used in the Error Analysis Section of the program

SYMBOL	CONTENTS	SYMBOL	CONTENTS
Q(4)	Ω_4 (Rad./min.)	Q(23)	a_{xN_8} (dimensionless)
Q(5)	Ω_5 "	Q(24)	a_{xN_9} "
Q(6)	ω_1 "	Q(25)	$a_{xN_{10}}$ "
Q(7)	ω_2 "	Q(26)	$a_{xN_{11}}$ "
Q(8)	ω_3 "	Q(27)	a_{yN_1} "
Q(9)	ω_4 "	Q(28)	a_{yN_2} "
Q(10)	ω_5 "	Q(29)	a_{yN_3} "
Q(11)	Ω_6 (rad.)	Q(30)	a_{yN_4} "
Q(12)	Ω_7 "	Q(31)	a_{yN_5} "
Q(13)	Ω_8 "	Q(88)	a_{yN_6} "
Q(14)	Ω_9 "	Q(32)	a_{yN_7} "
Q(15)	Ω_{10} "	Q(33)	a_{yN_8} "
Q(16)	a_{xN_1} (dimensionless)	Q(34)	a_{yN_9} "
Q(17)	a_{xN_2} "	Q(35)	$a_{yN_{10}}$ "
Q(18)	a_{xN_3} "	Q(36)	$a_{yN_{11}}$ "
Q(19)	a_{xN_4} "	Q(37)	$a_{yN_{12}}$ "
Q(20)	a_{xN_5} "	Q(38)	$a_{yN_{13}}$ "
Q(21)	a_{xN_6} "	Q(39)	M_1 (rad/min.)
Q(22)	a_{xN_7} "	Q(40)	M_2 "

SYMBOL	CONTENTS		SYMBOL	CONTENTS
Q(41)	M_3	(rad./min)	Q(60)	u_5 (rad.)
Q(42)	π_1	"	Q(61)	u_6 "
Q(43)	π_2	"	Q(62)	r_1 (earth radii (E.R.))
Q(44)	π_3	"	Q(63)	r_2 "
Q(45)	π_4	"	Q(64)	r_3 "
Q(46)	π_5	"	Q(65)	$\dot{r}_1$ (E.R./k_e^{-1}min)
Q(47)	L_1	(rad.)	Q(66)	$\dot{r}_2$ "
Q(48)	L_2	"	Q(67)	$\dot{r}_3$ "
Q(49)	L_3	"	Q(68)	$\dot{r}_4$ "
Q(50)	L_4	"	Q(69)	$\dot{r}_5$ "
Q(51)	L_5	"	Q(70)	$\dot{r}_6$ "
Q(52)	L_6	"	Q(71)	$\dot{r}_7$ "
Q(53)	i_1	"	Q(72)	$\dot{r}_8$ "
Q(54)	i_2	"	Q(73)	$\dot{r}_9$ "
Q(55)	i_3	"	Q(74)	$r\dot{v}_1$ "
Q(56)	u_1	"	Q(75)	$r\dot{v}_2$ "
Q(57)	u_2	"	Q(76)	$r\dot{v}_3$ "
Q(58)	u_3	"	Q(77)	$r\dot{v}_4$ "
Q(59)	u_4	"	Q(78)	rv_5 "

SYMBOL	CONTENTS		SYMBOL	CONTENTS	
Q(79)	$r\dot{v}_6$	(E.R./k_e^{-1} min.)	RTEØSQ	$\sqrt{1-e_o^2}$	
Q(80)	$r\dot{v}_7$	"	RTESQ	$\sqrt{1-e^2}$	
Q(89)	$r\dot{v}_8$	"	RTESQ	$\sqrt{1-e_L^2}$	
Q(81)	Ω_{11}	(rad.)	RTP	$\sqrt{p_o}$	(E.R.)$^{\frac{1}{2}}$
Q(82)	Ω_{12}	"	RVDØT	$r\dot{v}$	(E.R./k_e^{-1} min)
Q(83)	Ω_{13}	"	RVDØT	$(r\dot{v})_k$	"
Q(84)	Ω_{14}	"	SINEØ	$\sin (E+\omega)$	
Q(85)	i_4	"	SINI	$\sin i_o$	
Q(86)	i_5	"	SINILI	$\sin i$	
Q(87)	i_6	"	SINILN	$\mathrm{SIN}\ i_{oL_n}$	
R	r	(E.R.)	SINILS	$\sin^2 i_{oL}$	
R	r_k	"	SIN2IL	$\sin 2i_{oL}$	
RDELT1	$r\Delta\theta_1$	"	SINNØD	$\sin \Omega$	
RDELT3	$r\Delta\theta_3$	"	SINØM	$\sin \omega_{so}$	
RDØT	$\dot{r}$	(E.R./k_e^{-1} min.)	SIN2ØM	$\sin 2\omega_{so}$	
RDØT	$\dot{r}_k$	"	SIN3ØM	$\sin 3\omega_{so}$	
RN	r_n	(E.R.)	SINU	$\sin u$	
RN	r_{kn}	"	SINUN	$\sin u_n$	
RTA	$\sqrt{a}$	(E.R.)$^{\frac{1}{2}}$			

SYMBOL	CONTENTS	
SIN2U	sin 2u	
SIN3U	sin 3u	
SIN4U	sin 4u	
SU	u	(rad.)
SU	u_K	"
TEND	$t - t_o$	(min.)
THGR	θ_{gr}	(rad.)
THGRØ	θ_{gr_o}	"
U	U	"
UØ	U_o	"
UM	u_n	"
UN	u_{kn}	"
UX	U_x	
UY	U_y	
UZ	U_z	
VX	V_x	
VY	V_y	
VZ	V_z	
WX	W_x	

SYMBOL	CONTENTS	
WY	W_y	
WZ	W_z	
X	x	(E.R.)
Y	y	"
Z	z	"
XDØT	$\dot{x}$	(E.R./k_e^{-1}min
YDØT	$\dot{y}$	"
ZDØT	$\dot{z}$	"
XIN	i_{kn}	(rad.)
XINCL	i_o	"
XINCLI	i	"
XINCLI	i_{oL}	"
XINCLL	i_L	"
XJ2	J_2	
XJ3	J_3	
XJ4	J_4	
XKE	k_e	
XKELSQ	$n_o \angle \pi$	
XL	L	(rad.)

SYMBOL	CONTENTS	
XLØ	L_o	(rad.)
XLSUBL	L_L	"
XMU	μ	(Earth Mass Function)
XMX	M_x	
XMY	M_y	
XMZ	M_z	
XNØ	n	(E.R./k_e^{-1}min.)
XNØDE	Ω	(rad.)
XNØDE	Ω_{sL}	"
XNØDEL	Ω_L	"
XNØDEN	Ω_{kn}	"
XNØDEØ	Ω_o	"
XNØDES	Ω_{so}	"
XNØDØT	$\frac{d\Omega}{dt}$	(rad./min.)
XNX	N_x	
XNY	N_y	
XNZ	N_z	

Air Force Systems Command, Electronic Systems Division, 496L System Project Office, L. G. Hanscom Field, Bedford, Mass.
Rpt. No. ESD-TDR-63-432 A GENERAL PERTURBATIONS DIFFERENTIAL CORRECTION PROGRAM (U). Tech. rpt., August 63, 147 p. incl. diagrams, tables.

Unclassified Report

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1. Satellites (Artificial)
2. Celestial Mechanics
3. Orbital Trajectories
4. Tracking
5. Programming (Computers)

I. 496L SPO
II. Contract AF19(628)-562
III. Aeronutronic, a Div. of Ford Motor Company, Newport Beach, California

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